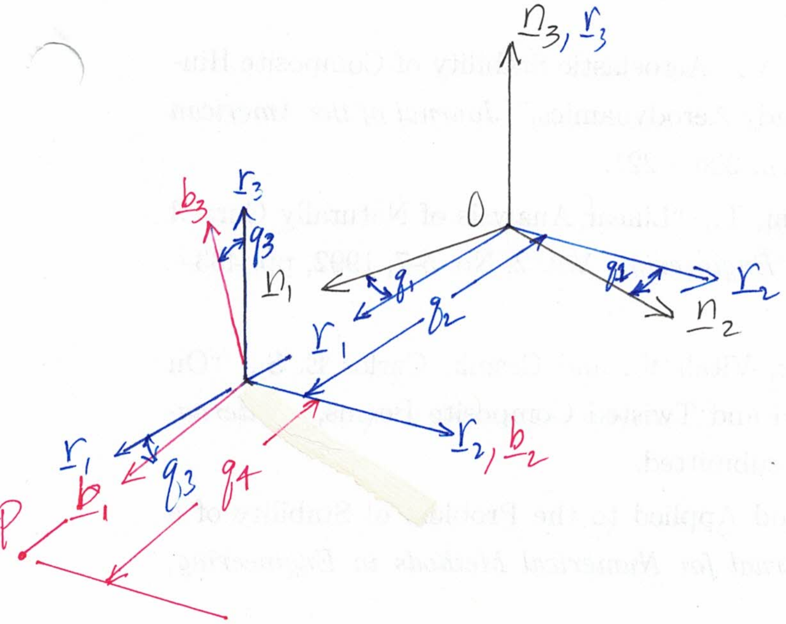


$$\underline{p} = q_2 \underline{r}_1 + q_4 \underline{b}_1$$

$$\underline{r}_1 = c_1 \underline{n}_1 + s_1 \underline{n}_2$$

$$\underline{r}_2 = -s_1 \underline{n}_1 + c_1 \underline{n}_2$$

$$= \underline{r}_3 \times \underline{r}_1 = \underline{n}_3 \times \underline{r}_1$$

$$\underline{b}_1 = c_3 \underline{r}_1 - s_3 \underline{r}_3$$

$$\underline{p} = q_2 \underline{r}_1 + q_4 (c_3 \underline{r}_1 - s_3 \underline{r}_3)$$

$$= (q_2 + q_4 c_3) \underline{r}_1 - q_4 s_3 \underline{r}_3$$

$$= \theta_1 \underline{r}_1 + \theta_2 \underline{r}_2 + \theta_3 \underline{r}_3$$

$$\theta_1 = q_2 + q_4 c_3$$

$$\theta_2 = 0$$

$$\theta_3 = -q_4 s_3$$

$$\underline{p} = \phi_1 \underline{n}_1 + \phi_2 \underline{n}_2 + \phi_3 \underline{n}_3$$

$$\begin{Bmatrix} \underline{r}_1 \\ \underline{r}_2 \\ \underline{r}_3 \end{Bmatrix} = \begin{bmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \underline{n}_1 \\ \underline{n}_2 \\ \underline{n}_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \underline{n}_1 \\ \underline{n}_2 \\ \underline{n}_3 \end{Bmatrix} = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \underline{r}_1 \\ \underline{r}_2 \\ \underline{r}_3 \end{Bmatrix}$$

$$\underline{p} = (q_2 + q_4 c_3) c_1 \underline{n}_1 + (q_2 + q_4 c_3) s_1 \underline{n}_2 - q_4 s_3 \underline{n}_3 = \phi_1 \underline{n}_1 + \phi_2 \underline{n}_2 + \phi_3 \underline{n}_3$$

$\underline{p}$ is a vector

$\begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{Bmatrix}$ measure numbers of $\underline{p}$ in N

neither is a vector!

$\begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}$ measure numbers of $\underline{p}$ in R

When a vector is represented as a column matrix, unless the frame is specified in which the vector is expressed, such representation is incomplete.

Now let's look at derivatives

$\underline{n}_1, \underline{n}_2, \underline{n}_3$ fixed in $N$

$\underline{r}_1, \underline{r}_2, \underline{r}_3$ fixed in $R$

$\underline{b}_1, \underline{b}_2, \underline{b}_3$ fixed in $B$

Thus $\dfrac{{}^N\partial \underline{n}_i}{\partial(\text{anything})} = 0$

$\dfrac{{}^R\partial \underline{r}_i}{\partial(\text{anything})} = 0$

etc.

use with chain rule

$$\frac{{}^N\partial \underline{p}}{\partial q_2} = c_1 \underline{n}_1 + s_1 \underline{n}_2 = \underline{r}_1$$

$\Rightarrow$ The unit vectors in terms of which the derivative of a vector is expressed may be different from those of the frame of reference in which the derivative is taken

$c_1, s_1, 0$ measure numbers ~~in~~ $N$ $\begin{Bmatrix} c_1 \\ s_1 \\ 0 \end{Bmatrix}$

$1, 0, 0$ measure numbers in $R$ $\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$

column matrix can be used to represent a vector <u>only if basis</u> is specified!

$$\frac{{}^R\partial \underline{p}}{\partial q_2} = \underline{r}_1 \quad \left( \text{same as } \frac{{}^N\partial \underline{p}}{\partial q_2} \right)$$

$$\frac{{}^N\partial \underline{p}}{\partial q_1} = (q_2 + q_4 c_3)(-s_1 \underline{n}_1 + c_1 \underline{n}_2) = (q_2 + q_4 c_3)\underline{r}_2$$

$$\frac{{}^R\partial \underline{p}}{\partial q_1} = 0$$

$$\frac{{}^R\partial}{\partial q_2} \frac{{}^N\partial \underline{p}}{\partial q_1} = \underline{r}_2$$

$$\frac{{}^N\partial}{\partial q_2} \frac{{}^R\partial \underline{p}}{\partial q_1} = 0$$

same but may not be

$$\frac{{}^N\partial}{\partial q_1} \frac{{}^R\partial \underline{p}}{\partial q_2} = \underline{r}_2$$

$$\frac{{}^R\partial}{\partial q_1} \frac{{}^N\partial \underline{p}}{\partial q_2} = 0$$

But example in text, pp. 14, 15 shows order may matter

march thru text