Georgia Institute of Technology
School of Aerospace Engineering
AE 6220: Rotorcraft Dynamics and Aeroelasticity
9:30 – 10:45 a.m., MW, Guggenheim 246
Portion taught by Prof. Dewey H. Hodges, Jan. 7 – Feb. 25, 2019

Prerequisites: You need to have some background in dynamics and structural dynamics. AE 6210 and 6230 (or equivalent) will be helpful.

Goals: To introduce students to the fundamentals of rotorcraft dynamics and aeroelasticity and provide necessary background to equip the student to do additional research and analysis in the field.

Topics: The outline below will be followed by Prof. Hodges in the first half of the course (Jan. 7 – Feb. 25) with the intent to cover as much of the material as time permits. Prof. Prasad will take over on Feb. 27 and will cover the material that he will post separately.

Text: No text is required, but I suggest that you acquire at least one of the books (Johnson, Bielawa, or Bramwell) in the bibliography below. Note that Johnson's older book, although dated, is now available in paperback from Dover. Johnson's newer book was only recently published. Finally, for those interested in mastering the details of rotor blade modeling, I suggest acquiring the book *Nonlinear Composite Beam Theory* by Hodges.

1. Introduction
    - 1.1. Overview of the course
    - 1.2. Review of techniques for derivation of equations of motion
        - 1.2.1. Notation and kinematical concepts
        - 1.2.2. Lagrange's equations
        - 1.2.3. Lagrange's form of D'Alembert's principle
        - 1.2.4. Hamilton's principle
        - 1.2.5. Principle of virtual work
    - 1.3. Review of dynamics of rotating bodies[1]
2. Stability of a Hingeless Rotor Blade with a Rigid-blade Model[2]
    - 2.1. Inertial forces
    - 2.2. Blade/root flexibility
    - 2.3. Behavior of the *in-vacuo* solution – homework
    - 2.4. Simple aerodynamic model
        - 2.4.1. lift and drag
        - 2.4.2. induced inflow velocity
        - 2.4.3. incorporation into rigid-blade equations
    - 2.5. Behavior of the stability from closed-form expression of Peters (1975)
    - 2.6. Behavior of the stability – homework

[1] See Bramwell *et al.* (2001), chapter 2, or Bielawa (2006), chapters 2 and 4.

[2] See Johnson (1980), article 12-3.3; Bramwell *et al.* (2001), chapter 11; or Bielawa (2006), chapter 10.

3. Linear Free Vibration of a Rotating Elastic Beam
    - 3.1. Internal forces
    - 3.2. Inertial forces
    - 3.3. Solution methods
        - 3.3.1. perturbation method
        - 3.3.2. transfer matrix method
        - 3.3.3. Ritz method
        - 3.3.4. collocation method
        - 3.3.5. mixed method
        - 3.3.6. finite element method
    - 3.4. Behavior of the solution – homework

4. Coupled Rotor/body Systems[3]
    - 4.1. Multi-blade coordinate transformation
        - 4.1.1. azimuthally periodic structure transformation
        - 4.1.2. rotating structure transformation
        - 4.1.3. use in a virtual work analysis
    - 4.2. A practical example: ground resonance
        - 4.2.1. example derivation of governing equations
        - 4.2.2. solution of governing equations – homework

5. Flexible-blade Modeling for Aeroelasticity[4]
    - 5.1. Hodges and Dowell – which many analysts still view as state of the art
        - 5.1.1. brief walk through the report
        - 5.1.2. nonlinear phenomena
            - 5.1.2.1. flap-lag coupling
            - 5.1.2.2. bending-torsion coupling
                - 5.1.2.2.1. Mil's terms
                - 5.1.2.2.2. controversy over torsion variables
                - 5.1.2.2.3. nonlinear GJ terms
            - 5.1.2.3. quasi-coordinates
                - 5.1.2.3.1. analogous to generalized speeds in dynamics
                - 5.1.2.3.2. discretize strains rather than displacements
                - 5.1.2.3.3. how they affect equations
                - 5.1.2.3.4. example
    - 5.2. Nonlinear kinematics of beams
        - 5.2.1. polar decomposition theorem
        - 5.2.2. local versus global deformation
    - 5.3. Elasticity
        - 5.3.1. solution of the local deformation problem
        - 5.3.2. warping: isotropic versus anisotropic
    - 5.4. External forces
        - 5.4.1. body forces, including inertial forces
        - 5.4.2. aerodynamic forces

[3] See Bramwell *et al.* (2001), chapter 12, or Bielawa (2006), chapter 11.

[4] See Hodges and Dowell (1974) and Hodges (2006).

- 5.5. Equilibrium equations
  - 5.5.1. intrinsic form
  - 5.5.2. solution of statics of a cantilever with tip mass
- 5.6. Overview of various special cases and theoretical issues
  - 5.6.1. Uncoupled equations
    - 5.6.1.1. lead-lag equation
      - 5.6.1.1.1. negative spring term
      - 5.6.1.1.2. fundamental frequency versus rotor angular speed
      - 5.6.1.1.3. what happened to the Coriolis term?
    - 5.6.1.2. torsion equation
      - 5.6.1.2.1. pretwist
      - 5.6.1.2.2. tennis racquet term
      - 5.6.1.2.3. trapeze term

Bibliography

Bielawa, Richard L.: *Rotary Wing Structural Dynamics and Aeroelasticity*, 2nd edition, AIAA, Reston, Virginia, 2006.

Bramwell, Anthony R. S.; Done, George T. S.; Balmford, David: *Bramwell's Helicopter Dynamics*, 2nd edition, Butterworth-Heinemann, Oxford, 2001.

Cesnik, Carlos E. S.; and Hodges, Dewey H.: "VABS: A New Concept for Composite Rotor Blade Cross-Sectional Modeling." *Journal of the American Helicopter Society*, vol. 42, no. 1, Jan. 1997, pp. 27 – 38.

Danielson, Donald A.; and Hodges, Dewey H.: "Nonlinear Beam Kinematics by Decomposition of the Rotation Tensor." *J. Applied Mechanics*, vol. 54, no. 2, 1987, pp. 258 – 262.

Danielson, Donald A.; and Hodges, Dewey H.: "A Beam Theory for Large Global Rotation, Moderate Local Rotation, and Small Strain." *J. Applied Mechanics*, vol. 55, no. 1, 1988, pp. 179 – 184.

Hodges, Dewey H.: "Torsion of Pretwisted Beams Due to Axial Loading." *J. Applied Mechanics*, vol. 47, no. 2, June 1980, pp. 393 – 397.

Hodges, Dewey H.: "A Mixed Variational Formulation Based on Exact Intrinsic Equations for Dynamics of Moving Beams." *International Journal of Solids and Structures*, vol. 26, no. 11, 1990, pp. 1253 – 1273.

Hodges, Dewey H.: *Nonlinear Composite Beam Theory*, AIAA, Reston, Virginia, 2006.

Hodges, Dewey H.; Atilgan, Ali R.; Cesnik, Carlos E. S.; and Fulton, Mark V.: "On a Simplified Strain Energy Function for Geometrically Nonlinear Behavior of Anisotropic Beams." *Composites Engineering*, vol. 2, nos. 5 – 7, 1992, pp. 513 – 526.

Hodges, Dewey H.; Chung, Yoon Y.; and Shang, Xiao Y.: "Discrete Transfer Matrix Method for Rotating Beams." *Journal of Sound and Vibration*, vol. 169, no. 2, Jan. 13, 1994, pp. 276 – 283.

Hodges, D. H.; and Dowell, E. H.: "Nonlinear Equations of Motion for the Elastic Bending and Torsion of Twisted Nonuniform Rotor Blades." NASA TN D-7818, 1974.

Hodges, Dewey H.; Ormiston, Robert A.; and Peters, David A.: "On the Nonlinear Deformation Geometry of Euler-Bernoulli Beams." NASA TP-1566, Apr. 1980 (Also AVRADCOM Technical Report 80-A-1).

Hodges, D. H.; Shang, X.; and Cesnik, C. E. S.: "Finite Element Solution of Nonlinear Intrinsic Equations for Curved Composite Beams," *Journal of the American Helicopter Society*, vol. 41, no. 4, Oct. 1996, pp. 313 – 321.

Johnson, Wayne: *Helicopter Theory*, Princeton University Press, Princeton, New Jersey, 1980 (also Dover Publications, Inc., New York, 1994).

Johnson, Wayne: *Rotorcraft Aeromechanics*, Cambridge University Press, Cambridge, UK, 2013.

Ormiston, Robert A.; and Hodges, Dewey H.: "Linear Flap-Lag Dynamics of Helicopter Rotor Blades in Hover." *J. American Helicopter Soc.*, vol. 17, no. 2, Apr. 1972, pp. 2 – 14.

Peters, David A.: "An Approximate Solution for the Free-Vibrations of Rotating Uniform Cantilever Beams." NASA TM-62,299, 1973.

Peters, David A.: "An Approximate Closed-form Solution for Lead-Lag Damping of Rotor Blades in Hover." NASA TM-62,425, 1975.

Shang, Xiaoyang; Hodges, Dewey H.; and Peters, David A.: "Aeroelastic Stability of Composite Hingeless Rotors in Hover with Finite-State Unsteady Aerodynamics," *Journal of the American Helicopter Society*, vol. 44, no. 3, July 1999, pp. 206 – 221.

Yu, Wenbin; Hodges, Dewey H.; Volovoi, Vitali, V.; and Cesnik, Carlos E. S.: "On Timoshenko-Like Modeling of Initially Curved and Twisted Composite Beams," *International Journal of Solids and Structures*, vol. 39, no. 19, 2002, pp. 5101 – 5121.