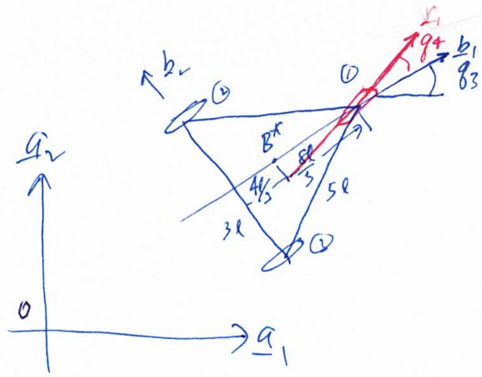

B 3 gc's
W_i 3 gc's
q_4 1 gc
7

$$r_{OB^*} = q_1 a_1 + q_2 a_2$$

$$^A v^{B^*} = u_1 b_1 + u_2 b_2 = \dot q_1 a_1 + \dot q_2 a_2$$

q_3, q_4 as shown

q_5, q_6, q_7 angles for W_1, W_2, W_3

3 wheeled vehicle rolls on $a_1 a_2$ plane with wheels of radius r

$$^A\omega^B = u_3 b_3 = \dot q_3 b_3$$

$n = 7$

$$^A\omega^{W_i} = {}^A\omega^B + {}^B\omega^{W_i}$$

$$^B\omega^{W_1} = u_4 b_3 + u_5 r_2 = \dot q_4 b_3 + \dot q_5 r_2$$

$$^B\omega^{W_2} = u_6 b_2 = \dot q_6 b_2$$

$$^B\omega^{W_3} = u_7 b_2 = \dot q_7 b_2$$

$$^A v^{W_1^*} = {}^A v^{B^*} + {}^A\omega^B \times r_{B^* W_1^*} = u_1 b_1 + u_2 b_2 + u_3 b_3 \times \frac{8l}{3} b_1$$
$$= u_1 b_1 + \left(u_2 + \frac{8l}{3} u_3\right) b_2$$

$$^A v^{W_2^*} = u_1 b_1 + u_2 b_2 + u_3 b_3 \times \left(-\frac{4l}{3} b_1 + 3l b_2\right)$$
$$= (u_1 - 3l u_3) b_1 + \left(u_2 - \frac{4l}{3} u_3\right) b_2$$

$$^A v^{W_3^*} = (u_1 + 3l u_3) b_1 + \left(u_2 - \frac{4l}{3} u_3\right) b_2$$

We can form holonomic partial velocities here, but they really are not needed since we know the system is constrained. Can write $^A v^{\hat W_i} = 0$ or (a bit easier) write $^A v^{W_i^*}$ another way using the fact that $^A v^{\hat W_i} = 0$.

$$^A v^{W_i^*} = \cancel{^A v^{\hat W_i}}^{\,0} + {}^A\omega^{W_i} \times r \underset{r_3}{b_3}$$

$$= (\cancel{u_3 b_3} + \cancel{u_4 b_3} + u_5 r_2) \times r r_3 = r u_5 r_1$$

$$^A v^{W_2^*} = (u_3 b_3 + u_6 b_2) \times r b_3 = r u_6 b_1$$

$$^A v^{W_3^*} = (u_3 b_3 + u_7 b_2) \times r b_3 = r u_7 b_1$$

Thus,
$$u_1 b_1 + \left(u_2 + \frac{8l}{3} u_3\right) b_2 = r u_5 r_1$$
$$(u_1 - 3l u_3) b_1 + \left(u_2 - \frac{4l}{3} u_3\right) b_2 = r u_6 b_1$$
$$(u_1 + 3l u_3) b_1 + \left(u_2 - \frac{4l}{3} u_3\right) b_2 = r u_7 b_1$$

Thus, $u_1 = r u_5 \, \underline{r}_1 \cdot \underline{b}_1 = r u_5 c_4$

$u_2 + \frac{8l}{3} u_3 = r u_5 \, \underline{r}_1 \cdot \underline{b}_2 = r u_5 s_4$

$u_1 - 3l u_3 = r u_6$

$u_1 + 3l u_3 = r u_7$

$u_2 - \frac{4l}{3} u_3 = 0$

$m = 5$

$p = n - m$

$\therefore p = 2$

Eliminate all but u_4 and u_1

$$\begin{bmatrix} 0 & 0 & rc_4 & 0 & 0 \\ 1 & \frac{8l}{3} & -rs_4 & 0 & 0 \\ 0 & 3l & 0 & r & 0 \\ 0 & -3l & 0 & 0 & r \\ 1 & -\frac{4l}{3} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \\ u_5 \\ u_6 \\ u_7 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{Bmatrix} u_1$$

$u_2 = \frac{u_1}{3} \tan q_4$

$u_3 = \frac{u_1}{4l} \tan q_4$

$u_5 = \frac{u_1}{rc_4}$

$u_6 = \frac{(4c_4 - 3s_4) u_1}{4rc_4}$

$u_7 = \frac{(4c_4 + 3s_4) u_1}{4rc_4}$

nonholonomic partial velocities + angular velocities

$${}^A\underline{\omega}^B = \frac{u_1}{4l} \tan q_4 \, \underline{b}_3 \qquad {}^A\tilde{\underline{\omega}}^B_1 = \frac{\tan q_4}{4l} \underline{b}_3$$

$${}^A\underline{\omega}^{W_1} = \left(\frac{u_1}{4l} \tan q_4 + u_4 \right) \underline{b}_3 + \frac{u_1}{rc_4} \underline{r}_2$$

$${}^A\underline{\omega}^{W_2} =$$

$${}^A\tilde{\underline{\omega}}^{W_1}_1 = \frac{\tan q_4}{4l} \underline{b}_3 + \frac{\underline{r}_2}{rc_4}$$

$${}^A\tilde{\underline{\omega}}^{W_1}_4 = \underline{b}_3$$

Kin. diff eq.

$$\dot{q}_1 = \left(u_1 \underline{b}_1 + \frac{u_1}{3} \tan q_4 \, \underline{b}_2 \right) \cdot \underline{a}_1 = u_1 c_3 - \frac{u_1}{3} \tan q_4 \, s_3$$

$$\dot{q}_2 = (\qquad) \cdot \underline{a}_2 = u_1 s_3 + \frac{u_1}{3} \tan q_4 \, c_3$$

$$\dot{q}_3 = \frac{u_1}{4l} \tan q_4$$

$$\dot{q}_4 = u_4$$

$$\dot{q}_5 = \frac{u_1}{rc_4}$$

$$\dot{q}_6 = \frac{u_1}{r}\left(1 - \frac{3}{4} \tan q_4\right)$$

$$\dot{q}_7 = \frac{u_1}{r}\left(1 + \frac{3}{4} \tan q_4\right)$$