Georgia Institute of Technology
School of Aerospace Engineering

Aerospace Engineering 4220

Introduction to Structural Dynamics and Aeroelasticity

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I certify that, in full accord with the Honor Code of the Georgia Institute of Technology, I have neither received assistance from nor given assistance to other students in taking this examination.

Problems: Closed book

1. Consider the uniform elastic beam shown in Fig. 1 undergoing bending oscillations. The beam has length ℓ, mass per unit length m, and flexural rigidity EI. The $x = 0$ end is pinned, and the $x = \ell$ end is restrained by a mechanism as shown in the figure. As shown in Fig. 1, there is a massless rigid rod of length $h = \ell\sigma$ that rotates with the right end cross section. At the end of this rigid rod is a particle of mass $m_c = \mu m\ell$, and it is restrained as shown by a light, linear spring having elastic constant $k = EI\kappa/\ell^3$.

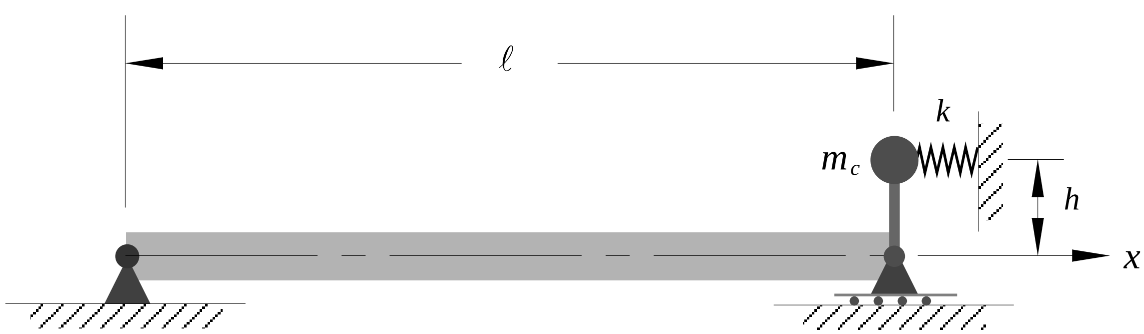

Figure 1: Schematic of beam with boundary spring and mass

(a) Write down all of the boundary conditions except the moment boundary condition on the right in terms of $v(x,t)$ and its partial derivatives. Show that the moment boundary condition at the right end can be written as

$$\beta_{xx}\frac{\partial^2 v}{\partial x^2}(\ell,t) + \beta_x\frac{\partial v}{\partial x}(\ell,t) + \beta_{xtt}\frac{\partial^3 v}{\partial x \partial t^2}(\ell,t) = 0$$

and find β_{xx}, β_x, and β_{xtt} in terms of EI, k, m_c, and h. (bonus: 5 points)

(b) Derive all of the boundary conditions except the moment boundary condition in terms of X and its derivatives. In particular, show that X vanishes at both ends, that the moment boundary condition at the left end is $X''(0) = 0$, and that at the right end the moment boundary condition is given by

$$\ell X''(\ell) + \sigma^2[\kappa - \mu(\alpha\ell)^4]X'(\ell) = 0$$

where α is the characteristic constant. (5 points)

(c) Show that the characteristic equation can be expressed as

$$2\alpha\ell\tan(\alpha\ell)\tanh(\alpha\ell) + \sigma^2[\kappa - \mu(\alpha\ell)^4][\tan(\alpha\ell) - \tanh(\alpha\ell)] = 0$$

(15 points)

(d) Show that the i^{th} mode shape can be written as

$$\phi_i(x) = \sin(\alpha_i x) + \beta_i \sinh(\alpha_i x)$$

and find β_i in terms of $\alpha_i\ell$ alone. (10 points)

(e) Suppose EI, m, and ℓ are fixed, $m_c = 0$ and $h = \ell/10$. Determine the value of k such that the fundamental frequency is $10\sqrt{EI/(m\ell^4)}$. (10 points)

2. A spanwise uniform, rigid wing of mass m, with a symmetrical airfoil, is pivoted about a spanwise axis which passes through the trailing edge. The motion of wing is restrained by a linearly elastic rotational spring, with spring constant k, attached to the trailing edge, as shown in Fig. 2. Both the pivot and the spring are attached to a rigid supporting structure and mounted in a wind tunnel with direction of the flow as shown. The sectional aerodynamic center coincides with the sectional center of gravity. The angle of attack is denoted by $\alpha = \alpha_r + \theta$, where α_r is the angle of attack when the spring is unstretched. Denote the freestream dynamic pressure by q, the planform area by S, and the lift-curve slope of the wing by $C_{L\alpha}$.

(a) Show that the value of dynamic pressure at which divergence occurs is given by

$$q_D = \frac{k}{eSC_{L\alpha}}$$

(15 points)

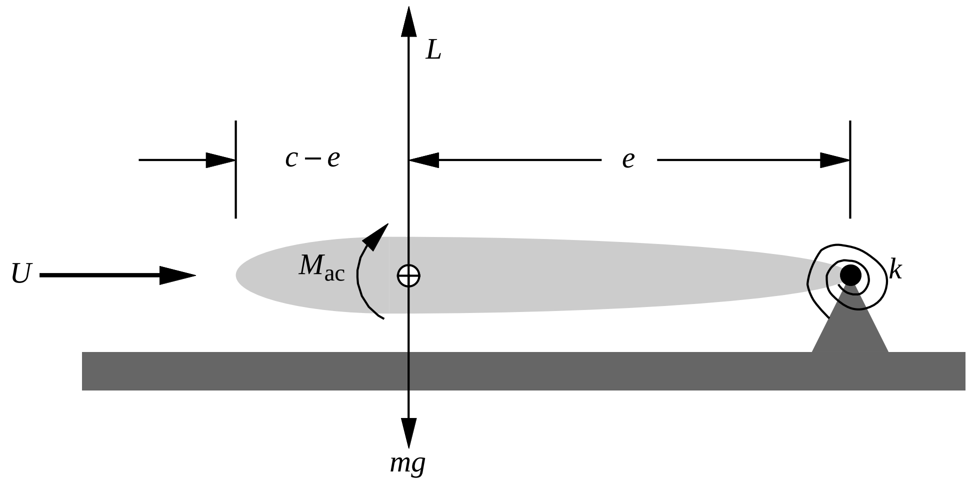

Figure 2: Schematic of rigid wing with trailing edge pivot and trailing edge rotational spring

(b) Show that the relative change in lift due to aeroelastic effects is given by

$$\frac{L - L_{\mathrm{rigid}}}{L_{\mathrm{rigid}}} = \frac{q - \frac{mg}{SC_{L\alpha}\alpha_{\mathrm{r}}}}{q_D - q}$$

Consider the sign of this relative change in lift for the case when $\alpha_{\mathrm{r}} > 0$ and

$$q < \frac{mg}{SC_{L\alpha}\alpha_{\mathrm{r}}}$$

Is the sign of the relative change in lift the same or different from the cases discussed in class and described in the text? (20 points)