# Georgia Institute of Technology
# School of Aerospace Engineering
# AE 6220: Rotorcraft Dynamics and Aeroelasticity

Prof. Dewey H. Hodges
Due: Feb. 1, 2019

Derive the equations of motion for the simple dynamical system described in class that exhibits the "ground resonance" type instability. In addition to the springs restraining the horizontal block motion and blade lead-lag motion, add a damper for the block motion with damping constant $C_x$ and a damper for each blade with damping constant $C_\zeta$. Hint: your linearized equations *before* applying the multi-blade coordinate transformation will each have one additional term. *After* you apply the transformation, the body equation will pick up only one term; whereas, the cyclic rotor equations will each pick up two. After a suitable scheme to make the equations dimensionless, solve for the eigenvalues and eigenvectors of the system as a function of $\overline{\Omega}$ for $\omega_x = 0.2$, $\omega_\zeta = 0.4$, $\overline{r} = 0.5$, and $\mu = .2$. Assume that the uncoupled critical damping ratios $\eta_x$ and $\eta_\zeta$ are small compared to unity and can be varied independently.

1. Plot the real and imaginary parts of the eigenvalues versus rotor angular speed for a few representative values of damping. How does the stability change as a function of damping? Does the product of damping $\eta_x\eta_\zeta$ have any special significance?

2. How much damping is required to stabilize the system? How does this value vary as $\mu$, $\omega_x$, and $\omega_\zeta$, change by factors of 2 in either direction?

3. How do the mode shapes qualitatively change as the rotor angular speed varies from small (the stiff-inplane regime) to moderate (the soft-inplane regime but smaller than the unstable value and also in the unstable regime) to large (larger than its value in the unstable regime)? Can you speculate on the reason why there is instability in one regime and not in others based on the behavior of the mode shapes? (Suggestion: let the phase of the $X$ degree of freedom, which is of course arbitrary, be zero. The phases of the other degrees of freedom are then easily seen. You may set damping equal to zero for this part.)

4. It is well known that damping may not always stabilize a system. One situation in which it may destabilize the system is when the damping for one degree of freedom is much larger than that in another. Explore some extreme values for $\eta_x/\eta_\zeta$ (say $10^{-4}$ or $10^4$) for the baseline values of other parameters, and see if this system can be destabilized by damping. In other words, if I increase the level of damping, the system will become less stable. Do you think the widely known "product-of-damping" criterion is valid? Run some cases to support your answer. (This criterion asserts that if the product $\eta_x\eta_\zeta$ is sufficiently large, then the system will be stable.)

I would not object to your comparing results for parts 1 and 2 with each other to assure that your codes are working correctly. However, please work independently on your investigation of parts 3 and 4. These parts of the problem will require some thought.