

$$\text{In}[1] := \mathbf{V} = \frac{1}{2} (k_{\xi h} \xi_h[\mathbf{t}]^2 + k_{\beta h} \beta_h[\mathbf{t}]^2 + k_{\xi b} \xi_b[\mathbf{t}]^2 + k_{\beta b} \beta_b[\mathbf{t}]^2)$$

General::spell1 : Possible spelling error: new symbol name "βh" is similar to existing symbol "ζh". [More...](#)

General::spell : Possible spelling error: new symbol name "ζb" is similar to existing symbols {ζ, ζh}. [More...](#)

General::spell : Possible spelling error: new symbol name "βb" is similar to existing symbols {β, βh, ζb}. [More...](#)

$$\text{Out}[1] = \frac{1}{2} (k_{\beta b} \beta_b(t)^2 + k_{\beta h} \beta_h(t)^2 + k_{\xi b} \xi_b(t)^2 + k_{\xi h} \xi_h(t)^2)$$

$$\text{In}[2] := k_{\xi b} = \frac{k_{\xi}}{R}$$

$$\text{Out}[2] = \frac{k_{\xi}}{R}$$

$$\text{In}[3] := k_{\beta b} = \frac{k_{\beta}}{R}$$

$$\text{Out}[3] = \frac{k_{\beta}}{R}$$

$$\text{In}[4] := k_{\xi h} = \frac{k_{\xi}}{1 - R}$$

$$\text{Out}[4] = \frac{k_{\xi}}{1 - R}$$

$$\text{In}[5] := k_{\beta h} = \frac{k_{\beta}}{1 - R}$$

$$\text{Out}[5] = \frac{k_{\beta}}{1 - R}$$

$$\text{In}[6] := \xi_b[\mathbf{t}] = \text{Cos}[\theta[\mathbf{t}]] (\xi[\mathbf{t}] - \xi_h[\mathbf{t}]) + \text{Sin}[\theta[\mathbf{t}]] (\beta[\mathbf{t}] - \beta_h[\mathbf{t}])$$

$$\text{Out}[6] = \sin(\theta(t)) (\beta(t) - \beta_h(t)) + \cos(\theta(t)) (\xi(t) - \xi_h(t))$$

$$\text{In}[7] := \beta_b[\mathbf{t}] = -\text{Sin}[\theta[\mathbf{t}]] (\xi[\mathbf{t}] - \xi_h[\mathbf{t}]) + \text{Cos}[\theta[\mathbf{t}]] (\beta[\mathbf{t}] - \beta_h[\mathbf{t}])$$

$$\text{Out}[7] = \cos(\theta(t)) (\beta(t) - \beta_h(t)) - \sin(\theta(t)) (\xi(t) - \xi_h(t))$$

$$\text{In}[8] := \mathbf{V} = \text{Simplify}[\mathbf{V}]$$

$$\text{Out}[8] = \frac{1}{2} \left(\frac{k_{\beta} \beta_h(t)^2}{1 - R} + \frac{k_{\xi} (\sin(\theta(t)) (\beta(t) - \beta_h(t)) + \cos(\theta(t)) (\xi(t) - \xi_h(t)))^2}{R} + \frac{k_{\beta} (\cos(\theta(t)) (\beta(t) - \beta_h(t)) - \sin(\theta(t)) (\xi(t) - \xi_h(t)))^2}{R} + \frac{k_{\xi} \xi_h(t)^2}{1 - R} \right)$$

$$\text{In}[9] := \text{eqzeta} = \text{Simplify}[\text{Expand}[\partial_{\xi_h}[\mathbf{t}] \mathbf{V}]]$$

$$\text{Out}[9] = \frac{1}{(R - 1)R}$$

$$((R - 1) \sin(\theta(t)) k_{\beta} (\cos(\theta(t)) \beta(t) - \sin(\theta(t)) \xi(t) - \cos(\theta(t)) \beta_h(t) + \sin(\theta(t)) \xi_h(t)) - k_{\xi} ((R - 1) \xi(t) \cos^2(\theta(t)) - R \xi_h(t) \cos^2(\theta(t)) + \xi_h(t) \cos^2(\theta(t)) + (R - 1) \sin(\theta(t)) \beta(t) \cos(\theta(t)) - R \sin(\theta(t)) \beta_h(t) \cos(\theta(t)) + \sin(\theta(t)) \beta_h(t) \cos(\theta(t)) + R \xi_h(t)))$$

In[10]:= **eqbeta = Simplify[Expand[$\partial_{\beta_h}[\mathbf{t}] \mathbf{V}$]]**

General::spell1 : Possible spelling error: new symbol name "eqbeta" is similar to existing symbol "eqzeta". **More...**

$$\text{Out[10]} = -\frac{1}{(R-1)R} ((R-1) \sin(\theta(t)) k_\zeta (\sin(\theta(t)) \beta(t) + \cos(\theta(t)) \zeta(t) - \sin(\theta(t)) \beta_h(t) - \cos(\theta(t)) \zeta_h(t)) + k_\beta ((R-1) \beta(t) \cos^2(\theta(t)) - R \beta_h(t) \cos^2(\theta(t)) + \beta_h(t) \cos^2(\theta(t)) - (R-1) \sin(\theta(t)) \zeta(t) \cos(\theta(t)) + R \sin(\theta(t)) \zeta_h(t) \cos(\theta(t)) - \sin(\theta(t)) \zeta_h(t) \cos(\theta(t)) + R \beta_h(t))$$

In[11]:= **sol = Solve[{eqzeta == 0, eqbeta == 0}, {\xi_h[t], \beta_h[t]}];**

In[12]:= **zrow = Simplify[\mathcal{D}_{\xi[t]} \mathbf{V} /. sol[[1]]];**

In[13]:= **zrowbcol = Simplify[\mathcal{D}_{\beta[t]} zrow]**

$$\text{Out[13]} = \frac{R \sin(2\theta(t)) k_\beta (k_\beta - k_\zeta) k_\zeta}{2((R-1)R \sin^2(\theta(t)) k_\beta^2 + (-R^2 + (R-1) \cos(2\theta(t)) R + R-1) k_\zeta k_\beta + (R-1)R \sin^2(\theta(t)) k_\zeta^2)}$$

In[14]:= **zrowbcol = zrowbcol /. Cos[2\theta[t]] -> 1 - 2 Sin[\theta[t]]^2**

$$\text{Out[14]} = \frac{R \sin(2\theta(t)) k_\beta (k_\beta - k_\zeta) k_\zeta}{2((R-1)R \sin^2(\theta(t)) k_\beta^2 + (-R^2 + (R-1)(1 - 2 \sin^2(\theta(t))) R + R-1) k_\zeta k_\beta + (R-1)R \sin^2(\theta(t)) k_\zeta^2)}$$

In[15]:= **zrowbcolnum = Numerator[zrowbcol]**

$$\text{Out[15]} = R \sin(2\theta(t)) k_\beta (k_\beta - k_\zeta) k_\zeta$$

$$\text{In[16]} := \Delta = -\frac{1}{2 k_\beta k_\zeta} (\text{Simplify}[\text{Denominator}[\text{zrowbcol}] /. \text{Cos}[2\theta[t]] -> 1 - 2 \text{Sin}[\theta[t]]^2])$$

$$\text{Out[16]} = -\frac{(R-1)R \sin^2(\theta(t)) k_\beta^2 - (2(R-1)R \sin^2(\theta(t)) + 1) k_\zeta k_\beta + (R-1)R \sin^2(\theta(t)) k_\zeta^2}{k_\beta k_\zeta}$$

In[17]:= **N[\Delta] /. {R -> .5, k_\beta -> .1, k_\zeta -> .5, \theta[t] -> .1}**

$$\text{Out[17]} = 1.00797$$

In[18]:= **\Delta = Expand[\Delta]**

$$\text{Out[18]} = 2R^2 \sin^2(\theta(t)) - 2R \sin^2(\theta(t)) - \frac{R^2 k_\zeta \sin^2(\theta(t))}{k_\beta} + \frac{R k_\zeta \sin^2(\theta(t))}{k_\beta} - \frac{R^2 k_\beta \sin^2(\theta(t))}{k_\zeta} + \frac{R k_\beta \sin^2(\theta(t))}{k_\zeta} + 1$$

$$\text{In[19]} := \text{Simplify}\left[\frac{\Delta - 1}{R(1 - R)}\right]$$

$$\text{Out[19]} = \frac{\sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta}$$

In[20]:= **\Delta = 1 + R(1 - R) %**

$$\text{Out[20]} = \frac{(1 - R)R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1$$

In[21]:= **N[\Delta] /. {R -> .5, k_\beta -> .1, k_\zeta -> .5, \theta[t] -> .1}**

$$\text{Out[21]} = 1.00797$$

$$\text{In}[22]:= \text{zrowbcolnum} = - \frac{\text{Numerator}[\text{zrowbcol}]}{2 k_{\beta} k_{\zeta}}$$

$$\text{Out}[22]= -\frac{1}{2} R \sin(2 \theta(t)) (k_{\beta} - k_{\zeta})$$

$$\text{In}[23]:= \text{zrowbcol} = \frac{\text{zrowbcolnum}}{\Delta}$$

$$\text{Out}[23]= -\frac{R \sin(2 \theta(t)) (k_{\beta} - k_{\zeta})}{2 \left(\frac{(1-R) R \sin^2(\theta(t)) (k_{\beta} - k_{\zeta})^2}{k_{\beta} k_{\zeta}} + 1 \right)}$$

$$\text{In}[24]:= \text{dzrowbcoldtheta} = \text{Simplify}[\partial_{\theta[t]} \text{zrowbcol}]$$

$$\text{Out}[24]= \frac{R \left(-\frac{(R-1) R \sin^2(2 \theta(t)) (k_{\beta} - k_{\zeta})^2}{2 k_{\beta} k_{\zeta}} - \cos(2 \theta(t)) \left(1 - \frac{(R-1) R \sin^2(\theta(t)) (k_{\beta} - k_{\zeta})^2}{k_{\beta} k_{\zeta}} \right) \right) (k_{\beta} - k_{\zeta})}{\left(\frac{(R-1) R \sin^2(\theta(t)) (k_{\beta} - k_{\zeta})^2}{k_{\beta} k_{\zeta}} - 1 \right)^2}$$

$$\text{In}[25]:= \mathbf{N}[\text{dzrowbcoldtheta}] /. \{R \rightarrow .5, k_{\beta} \rightarrow .1, k_{\zeta} \rightarrow .5, \theta[t] \rightarrow .1\}$$

$$\text{Out}[25]= 0.191355$$

$$\text{In}[26]:= \text{numdzrowbcoldtheta} = \text{Numerator}[\text{dzrowbcoldtheta}] / R / (k_{\beta} - k_{\zeta})$$

$$\text{Out}[26]= -\frac{(R-1) R \sin^2(2 \theta(t)) (k_{\beta} - k_{\zeta})^2}{2 k_{\beta} k_{\zeta}} - \cos(2 \theta(t)) \left(1 - \frac{(R-1) R \sin^2(\theta(t)) (k_{\beta} - k_{\zeta})^2}{k_{\beta} k_{\zeta}} \right)$$

$$\text{In}[27]:= \text{numdzrowbcoldtheta} = \text{Expand}[\text{numdzrowbcoldtheta} /. \{\text{Cos}[2 \theta[t]] \rightarrow 1 - 2 \text{Sin}[\theta[t]]^2, \text{Sin}[2 \theta[t]] \rightarrow 2 \text{Sin}[\theta[t]] \text{Cos}[\theta[t]]\}]$$

$$\begin{aligned} \text{Out}[27]= & 4 R^2 \sin^4(\theta(t)) - 4 R \sin^4(\theta(t)) - \frac{2 R^2 k_{\zeta} \sin^4(\theta(t))}{k_{\beta}} + \frac{2 R k_{\zeta} \sin^4(\theta(t))}{k_{\beta}} - \frac{2 R^2 k_{\beta} \sin^4(\theta(t))}{k_{\zeta}} + \\ & \frac{2 R k_{\beta} \sin^4(\theta(t))}{k_{\zeta}} - 2 R^2 \sin^2(\theta(t)) + 4 R^2 \cos^2(\theta(t)) \sin^2(\theta(t)) - 4 R \cos^2(\theta(t)) \sin^2(\theta(t)) + 2 R \sin^2(\theta(t)) + \\ & \frac{R^2 k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} - \frac{2 R^2 \cos^2(\theta(t)) k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} + \frac{2 R \cos^2(\theta(t)) k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} - \frac{R k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} + \\ & \frac{R^2 k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} - \frac{2 R^2 \cos^2(\theta(t)) k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} + \frac{2 R \cos^2(\theta(t)) k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} - \frac{R k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} + 2 \sin^2(\theta(t)) - 1 \end{aligned}$$

$$\text{In}[28]:= \text{numdzrowbcoldtheta} = \text{Expand}[\text{numdzrowbcoldtheta} /. \text{Cos}[\theta[t]]^2 \rightarrow 1 - \text{Sin}[\theta[t]]^2]$$

$$\text{Out}[28]= 2 R^2 \sin^2(\theta(t)) - 2 R \sin^2(\theta(t)) - \frac{R^2 k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} + \frac{R k_{\zeta} \sin^2(\theta(t))}{k_{\beta}} - \frac{R^2 k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} + \frac{R k_{\beta} \sin^2(\theta(t))}{k_{\zeta}} + 2 \sin^2(\theta(t)) - 1$$

$$\text{In}[29]:= \text{numdzrowbcoldthetap1} = \text{Simplify}[\text{numdzrowbcoldtheta} + 1]$$

$$\text{Out}[29]= -\frac{\sin^2(\theta(t)) ((R-1) R k_{\beta}^2 - 2(R^2 - R + 1) k_{\zeta} k_{\beta} + (R-1) R k_{\zeta}^2)}{k_{\beta} k_{\zeta}}$$

$$\text{In}[30]:= \text{Simplify}[\text{numdzrowbcoldthetap1} / \text{Sin}[\theta[t]]^2]$$

$$\text{Out}[30]= -\frac{(R-1) R k_{\beta}^2 - 2(R^2 - R + 1) k_{\zeta} k_{\beta} + (R-1) R k_{\zeta}^2}{k_{\beta} k_{\zeta}}$$

In[31]:= **Simplify**[% - 2]

$$\text{Out}[31]= -\frac{(R-1)R(k_\beta - k_\zeta)^2}{k_\beta k_\zeta}$$

In[32]:= $\frac{((2 + \%) \text{Sin}[\theta[t]]^2 - 1) R (k_\beta - k_\zeta)}{\Delta^2}$

$$\text{Out}[32]= \frac{R \left(\sin^2(\theta(t)) \left(2 - \frac{(R-1)R(k_\beta - k_\zeta)^2}{k_\beta k_\zeta} \right) - 1 \right) (k_\beta - k_\zeta)}{\left(\frac{(1-R)R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1 \right)^2}$$

In[33]:= **N**[%] /. {**R** -> .5, **k_β** -> .1, **k_ζ** -> .5, **θ[t]** -> .1}

Out[33]= 0.191355

In[34]:= **zrowzcol** = **∂_{g[t]}** **zrow**;

General::spell1 : Possible spelling error: new symbol name "zrowzcol" is similar to existing symbol "zrowbcol". **More...**

In[35]:= **zrowzcol** = **Simplify**[**zrowzcol**];

In[36]:= **zrowzcolnum** = - $\frac{\text{Numerator}[\text{zrowzcol}]}{2 k_\beta k_\zeta}$

General::spell1 : Possible spelling error: new symbol name "zrowzcolnum" is similar to existing symbol "zrowbcolnum". **More...**

$$\text{Out}[36]= \frac{1}{2} (2 R k_\beta \sin^2(\theta(t)) + (\cos(2 \theta(t)) R - R + 2) k_\zeta)$$

In[37]:= **zrowzcolnum** = **Expand**[**zrowzcolnum** /. **Cos**[2 **θ[t]**] -> 1 - 2 **Sin**[**θ[t]**]²]

$$\text{Out}[37]= R k_\beta \sin^2(\theta(t)) - R k_\zeta \sin^2(\theta(t)) + k_\zeta$$

In[38]:= **zrowzcol** = $\frac{\text{zrowzcolnum}}{\Delta}$

$$\text{Out}[38]= \frac{R k_\beta \sin^2(\theta(t)) - R k_\zeta \sin^2(\theta(t)) + k_\zeta}{\frac{(1-R)R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1}$$

In[39]:= **dzrowzcoldtheta** = **Simplify**[**∂_{θ[t]}** **zrowzcol**]

General::spell1 : Possible spelling error: new symbol name "dzrowzcoldtheta" is similar to existing symbol "dzrowbcoldtheta". **More...**

$$\text{Out}[39]= \frac{R \sin(2 \theta(t)) k_\beta (k_\beta - k_\zeta) k_\zeta^2 (R k_\beta - (R - 1) k_\zeta)}{((R - 1) R \sin^2(\theta(t)) k_\beta^2 + (-R^2 + (R - 1) \cos(2 \theta(t)) R + R - 1) k_\zeta k_\beta + (R - 1) R \sin^2(\theta(t)) k_\zeta^2)^2}$$

In[40]:= **N**[**dzrowzcoldtheta**] /. {**R** -> .5, **k_β** -> .1, **k_ζ** -> .5, **θ[t]** -> .1}

Out[40]= -0.117323

In[41]:= **numdzrowzcoldtheta** = **Simplify**[Δ^2 **dzrowzcoldtheta** / **R** / (**k_β** - **k_ζ**)]

General::spell1 : Possible spelling error: new symbol name "numdzrowzcoldtheta" is similar to existing symbol "numdzrowbcoldtheta". **More...**

$$\text{Out}[41]= \frac{\sin(2 \theta(t)) (R k_\beta - (R - 1) k_\zeta)}{k_\beta}$$

$$\text{In}[42] := \frac{\mathbf{R} (\mathbf{k}_\beta - \mathbf{k}_\zeta)}{\Delta^2} \%$$

$$\text{Out}[42] = \frac{R \sin(2 \theta(t)) (k_\beta - k_\zeta) (R k_\beta - (R - 1) k_\zeta)}{k_\beta \left(\frac{(1-R) R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1 \right)^2}$$

$$\text{In}[43] := \mathbf{N}[\%] /. \{\mathbf{R} \rightarrow .5, \mathbf{k}_\beta \rightarrow .1, \mathbf{k}_\zeta \rightarrow .5, \theta[\mathbf{t}] \rightarrow .1\}$$

$$\text{Out}[43] = -0.117323$$

$$\text{In}[44] := \mathbf{brow} = \mathbf{Simplify}[\partial_{\beta[\mathbf{t}]} \mathbf{V} /. \mathbf{sol}[[1]]];$$

General::spell1 : Possible spelling error: new symbol name "brow" is similar to existing symbol "zrow". **More...**

$$\text{In}[45] := \mathbf{browbcol} = \partial_{\beta[\mathbf{t}]} \mathbf{brow};$$

General::spell1 : Possible spelling error: new symbol name "browbcol" is similar to existing symbol "zrowbcol". **More...**

$$\text{In}[46] := \mathbf{browbcol} = \mathbf{Simplify}[\mathbf{browbcol}];$$

$$\text{In}[47] := \mathbf{browbcolnum} = - \frac{\mathbf{Numerator}[\mathbf{browbcol}]}{2 \mathbf{k}_\beta \mathbf{k}_\zeta}$$

General::spell1 : Possible spelling error: new symbol name "browbcolnum" is similar to existing symbol "zrowbcolnum". **More...**

$$\text{Out}[47] = \frac{1}{2} (2 R k_\zeta \sin^2(\theta(t)) + (\cos(2 \theta(t)) R - R + 2) k_\beta)$$

$$\text{In}[48] := \mathbf{browbcolnum} = \mathbf{Expand}[\mathbf{browbcolnum} /. \mathbf{Cos}[2 \theta[\mathbf{t}]] \rightarrow 1 - 2 \mathbf{Sin}[\theta[\mathbf{t}]]^2]$$

$$\text{Out}[48] = -R k_\beta \sin^2(\theta(t)) + R k_\zeta \sin^2(\theta(t)) + k_\beta$$

$$\text{In}[49] := \mathbf{browbcol} = \frac{\mathbf{browbcolnum}}{\Delta}$$

$$\text{Out}[49] = \frac{-R k_\beta \sin^2(\theta(t)) + R k_\zeta \sin^2(\theta(t)) + k_\beta}{\frac{(1-R) R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1}$$

$$\text{In}[50] := \mathbf{dbrowbcoltheta} = \partial_{\theta[\mathbf{t}]} \mathbf{browbcol}$$

General::spell1 : Possible spelling error: new symbol name "dbrowbcoltheta" is similar to existing symbol "dzrowbcoltheta". **More...**

$$\text{Out}[50] = \frac{2 R \cos(\theta(t)) \sin(\theta(t)) k_\zeta - 2 R \cos(\theta(t)) \sin(\theta(t)) k_\beta}{\frac{(1-R) R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1} - \frac{2 (1 - R) R \cos(\theta(t)) \sin(\theta(t)) (k_\beta - k_\zeta)^2 (-R k_\beta \sin^2(\theta(t)) + R k_\zeta \sin^2(\theta(t)) + k_\beta)}{k_\beta \left(\frac{(1-R) R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} + 1 \right)^2 k_\zeta}$$

$$\text{In}[51] := \mathbf{N}[\mathbf{dbrowbcoltheta}] /. \{\mathbf{R} \rightarrow .5, \mathbf{k}_\beta \rightarrow .1, \mathbf{k}_\zeta \rightarrow .5, \theta[\mathbf{t}] \rightarrow .1\}$$

$$\text{Out}[51] = 0.0234646$$

In[52]:= **numdbrowbcoldtheta = Simplify[Δ^2 dbrowbcoldtheta / R / (k $_{\beta}$ - k $_{\zeta}$)]**

General::spell1 : Possible spelling error: new symbol name "numdbrowbcoldtheta" is similar to existing symbol "numdzrowbcoldtheta". **More...**

$$\text{Out}[52]= \frac{\sin(2\theta(t))((R-1)k_{\beta}-Rk_{\zeta})}{k_{\zeta}}$$

In[53]:= **$\frac{R(k_{\beta}-k_{\zeta})}{\Delta^2}$ %**

$$\text{Out}[53]= \frac{R\sin(2\theta(t))(k_{\beta}-k_{\zeta})((R-1)k_{\beta}-Rk_{\zeta})}{\left(\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1\right)^2 k_{\zeta}}$$

In[54]:= **N[%] /. {R -> .5, k $_{\beta}$ -> .1, k $_{\zeta}$ -> .5, θ [t] -> .1}**

Out[54]= 0.0234646

In[55]:= **R $_w$ = Simplify[$\frac{1}{\Delta}$ $\partial_{\theta[t]}$ Δ]**

$$\text{Out}[55]= \frac{2(1-R)R\cos(\theta(t))\sin(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}\left(1-\frac{(R-1)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}\right)k_{\zeta}}$$

In[56]:= **N[R $_w$] /. {R -> .5, k $_{\beta}$ -> .1, k $_{\zeta}$ -> .5, θ [t] -> .1}**

Out[56]= 0.157678

In[57]:= **Denominator[R $_w$]**

$$\text{Out}[57]= k_{\beta}\left(1-\frac{(R-1)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}\right)k_{\zeta}$$

In[58]:= **N[%] /. {R -> .5, k $_{\beta}$ -> .1, k $_{\zeta}$ -> .5, θ [t] -> .1}**

Out[58]= 0.0503987

In[59]:= **k $_{\beta}$ k $_{\zeta}$ Δ**

$$\text{Out}[59]= k_{\beta}\left(\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1\right)k_{\zeta}$$

In[60]:= **N[%] /. {R -> .5, k $_{\beta}$ -> .1, k $_{\zeta}$ -> .5, θ [t] -> .1}**

Out[60]= 0.0503987

In[61]:= **(*Stiffness matrix w/o θ_{ζ} *)**

In[62]:= **kmatpe0 = {{zrowzcol, zrowbcol}, {zrowbcol, browbcol}}**

$$\text{Out}[62]= \begin{pmatrix} \frac{Rk_{\beta}\sin^2(\theta(t))-Rk_{\zeta}\sin^2(\theta(t))+k_{\zeta}}{\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1} & -\frac{R\sin(2\theta(t))(k_{\beta}-k_{\zeta})}{2\left(\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1\right)} \\ -\frac{R\sin(2\theta(t))(k_{\beta}-k_{\zeta})}{2\left(\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1\right)} & \frac{-Rk_{\beta}\sin^2(\theta(t))+Rk_{\zeta}\sin^2(\theta(t))+k_{\beta}}{\frac{(1-R)R\sin^2(\theta(t))(k_{\beta}-k_{\zeta})^2}{k_{\beta}k_{\zeta}}+1} \end{pmatrix}$$

In[65]:= **kmatpe0 = FullSimplify**[{{zrowzcol, 0}, {zrowbcol, 0}}]

$$\text{Out}[65]= \begin{pmatrix} \frac{R(k_\beta - k_\zeta) \sin^2(\theta(t)) + k_\zeta}{1 - \frac{(R-1)R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta}} & 0 \\ -\frac{R \sin(2\theta(t)) (k_\beta - k_\zeta)}{2 \left(1 - \frac{(R-1)R \sin^2(\theta(t)) (k_\beta - k_\zeta)^2}{k_\beta k_\zeta} \right)} & 0 \end{pmatrix}$$

In[66]:= **Clear**[Δ]

In[95]:= **kmatpe0**[**[1, 1, 1, 1]**] = Δ

Out[95]= Δ

In[96]:= **kmatpe0**[**[2, 1, 4, 1]**] = Δ

Out[96]= Δ

In[98]:= **kmatpe0** /. {**k $_\zeta$** \rightarrow ω_ζ^2 , **k $_\beta$** \rightarrow ω_β^2 }

$$\text{Out}[98]= \begin{pmatrix} \frac{R(\omega_\beta^2 - \omega_\zeta^2) \sin^2(\theta(t)) + \omega_\zeta^2}{\Delta} & 0 \\ -\frac{R \sin(2\theta(t)) (\omega_\beta^2 - \omega_\zeta^2)}{2\Delta} & 0 \end{pmatrix}$$