Errata for First Printing of *Fundamentals of Structural Stability*

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Page	Description
4	The first sentence in section 1.2.1 should read, "A mechanical system is conservative if subjected only to conservative forces." (The word "only" is missing.)
4	The **Parenthesis** at the bottom of the page should start out as "If S denotes a surface in space bounded by a space curve C, then. . . ."
5	In the line just below Eq. (4), the "and" should not be there.
6	A period is missing between "systems" and "Ziegler (1968)" near the end of section 1.2.1.
38	The Q and the associated arrow should be removed from Fig. 2.11.
41	The Q and the associated arrow should be removed from Fig. P2.1.
49	The ε in Eqs. (3), in the two unnumbered equations before Eqs. (3), and in the text just above them should be replaced with ϵ to be consistent with Fig. 3.1.
51	In the line just below Eq. (10), κ_{xx} should be replaced by k_{xx}.
56	In the text above the bottom two equations, replace R_1 with R_0.
59	Eq. (19) should be exactly as it appears on page 53, i.e. the x's should not be subscripted.
62	The last equation on this page (unnumbered) should be $$w(0) = 0, \quad w_{,x}(L) = 0$$
62	The symbol φ in all the text and equations on this page should be replaced by ϕ (to be consistent with the symbol in Fig. 3.10.

63 The symbol φ in all the text and equations on this page should be replaced by ϕ (to be consistent with the symbol in Fig. 3.10.

67 The symbol d in the left part of Fig. 3.12 (c) should be a.

68 The 3^{rd} line after Eq. (61), should start with "n=1.2" instead of "n=1,2."

69 The parenthetical term in Eq. (66) multiplied by P should be $\gamma + \psi$ instead of $\gamma - \psi$.

70 The text above Eq. (75) should be "...into Eq. (73) yields" instead of Eq. (80).

73 The text above Eq. (98), should be "...and the use of Eqs. (97b-97e) leads to...."

83 Just before Eq. (136) should read, "...elements of the first column of C^T can be expressed as"

83 Immediately following Eq. (136), delete "for small strain."

83 Text starting with the last sentence before Eq. (139) until the start of Eqs. (140) should read, "Starting with Eqs. (123), (133), (134) and (136), together with Eqs. (138) to approximate C' as

$$C' = \left(I - \widetilde{\hat{\theta}} + \frac{1}{2}\widetilde{\hat{\theta}}\widetilde{\hat{\theta}} \right) \left(\overline{C}\widetilde{k} - \widetilde{\overline{K}\,\overline{C}} \right) - \left(\widetilde{\hat{\theta}'} - \frac{1}{2}\widetilde{\hat{\theta}'}\widetilde{\hat{\theta}} - \frac{1}{2}\widetilde{\hat{\theta}}\widetilde{\hat{\theta}'} \right) \overline{C} \tag{140}$$

one can obtain approximate expressions valid through second order for $\hat{\epsilon}$, $\hat{\theta}_2$, $\hat{\theta}_3$, and $\hat{\kappa}$, given by"

83 The first of Eqs. (140) should be

$$\hat{\varepsilon} = (1 + \overline{\varepsilon}) \left[\hat{\phi}_1 + \frac{1}{2} \left(\hat{\phi}_2^2 + \hat{\phi}_3^2 \right) \right]$$

83 The last of Eqs. (140) should be

$$\hat{\kappa} = \left(I - \frac{1}{2}\widetilde{\hat{\theta}} \right) \left(\hat{\theta}' + \widetilde{\overline{K}}\hat{\theta} \right)$$

84 Eq. (141) should be

$$\hat{\phi} = \frac{\overline{C}(\hat{u}' + \widetilde{k}\hat{u})}{1 + \overline{\varepsilon}}$$

86 At the end of the paragraph that starts, "Four beam (1-D) variables..." should read, "...midsurface, and vectors are denoted with bold letters."

86 In Fig. 3.19, b_3 should be $\mathbf{b_3}$.

86 At the lower-left corner of Fig. 3.19, x_2 should be replaced by x_1.

86 In Fig. 3.19, n should be $\mathbf{n}$.

86 In Fig. 3.19, τ should be $\boldsymbol{\tau}$.

88 Insert immediately following Eqs. (157): "where $(\ )_{,1} \equiv \partial(\ )/\partial x_1 \approx O(1/\ell)$, $(\ )_{,2} \equiv \partial(\ )/\partial s \approx O(1/a)$."

88 The v_2 at the lower right portion of Fig. 3.20 should be u_2.

89 The first line should read, "depend on u_2' and u_3', the rigid-body rotations..."

89 Just before Eq. (160), "circumferential coordinate" should be replaced by "the contour coordinate."

89 Insert the following paragraph at the very end of the section entitled "Shell Theory": "At this point we may substitute the expressions for 2-D shell displacements in terms of 1-D beam displacements and the warping functions into the 2-D shell strain measures. These tedious expressions can be greatly simplified by making use of Eqs. (153) and the small parameters. Details can be found in Hodges (2006)."

89 Eq. (163) should be

$$A = 2h\mu \begin{bmatrix} 1+\sigma & 0 & \sigma \\ 0 & \frac{1}{2} & 0 \\ \sigma & 0 & 1+\sigma \end{bmatrix}$$

90 The first sentence after Eqs. (167) should be replaced by these two sentences: "Thus, the assumption of Eqs. (166) is seen to be equivalent to minimizing the energy with respect to unknowns $2\gamma_{12}$, γ_{22} and ρ_{22}, dominant terms of which contain only warping displacements and their partial derivatives with respect to s. Dominant terms of the other shell strains, i.e. γ_{11}, ρ_{11}, and ρ_{12}, contain only the 1-D beam variables u_i and θ and their derivatives."

90 The $\overline{B}$ in the lower left corner of the matrix on the right-hand side of Eq. (168) should be replaced with $-\overline{B}^T$.

90 In Eq. (172) the definition for Γ should have an h in front of it, i.e. $\Gamma = h\int_S \eta^2 ds$.

91 Replace "For a general cross section the strain energy can still be written..." with "Choosing x_2 and x_3 to be principal axes for the cross-section but without restricting it to be doubly-symmetric, we can still write the strain energy...."

92 Just before Eqs. (177), replace "... an equation of equilibrium of the form" with "... identification of an equivalent distributed load per unit area Q_2 such that"

92 Replace Fig. 3.22 with Fig. 1.

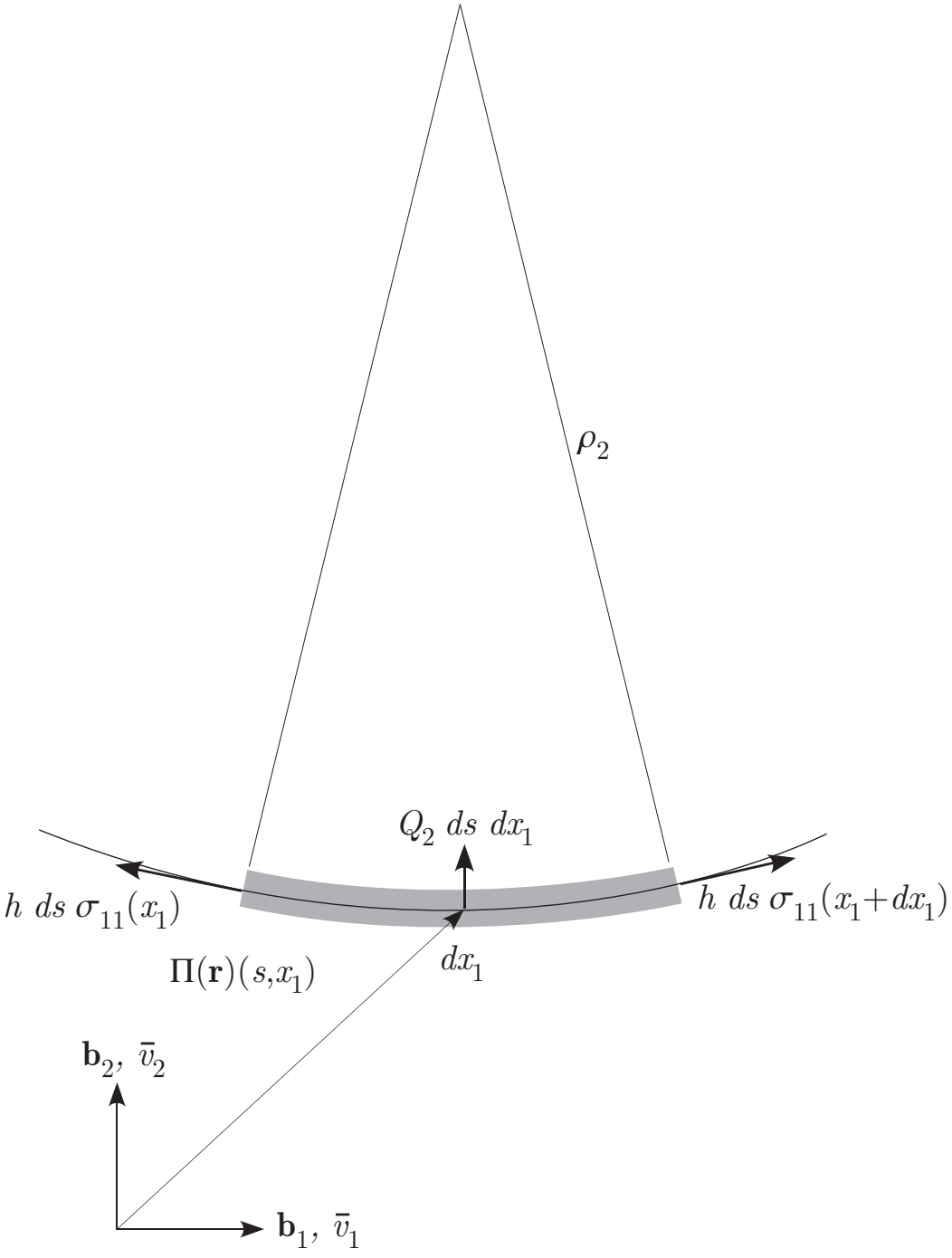

Figure 1: Replacement for Fig. 3.22

93 The three terms in Eqs. (182) with triple primes should all be quadruple primes.

94 Replace "cruciform" with "doubly symmetric" at the top of the page and after Eq. (183).

94 The sentence starting the paragraph just before Eq. (185) should begin, "For example, for a restraint at the boundary…."

94 At the very end of the section, before the problems, add: "It should be noted that, for a cruciform section such as shown in Fig. 3.23, Γ is very small, so that the critical load becomes independent of the column length as in Problem 3.17."

100 Problem 13(a) should use ℓ throughout the problem statement.

100 Problem 16 should read, "…show that $\rho_{12} = -\theta'$ and that, for isotropic materials, $\overline{A}_{11} = hE$, …."

100 For Problem 17, the last two sentences should read, "Assume one end is clamped and the other pinned; warping may be neglected. Determine the values of the ratio b/a…."

111 The displayed function in Fig. 4.6b should be identical to the right side of Eq. (46), i.e.

$$\frac{u}{1 + \frac{2EI}{\beta L}u^2} \tag{141}$$

120 The three references to Eq. (97) should all be Eq. (100).

121 The three references to Eq. (97) should all be Eq. (100).

122 The two references to Eq. (97) should both be Eq. (100).

123 The two references to Eq. (97) should both be Eq. (100).

124 The reference to Eq. (97) should be Eq. (100).

142 In Fig. 4.4a, the upper set of rollers should have a support to its right exactly as the upper support shows in Fig. 4.4b.

142 Figure 4.5 was incorrectly repeated on this page and called Fig. P4.5. Actually, Fig. P4.5 should be as shown in Fig. 2.

165 The upper left of the unnumbered equations at the bottom of the page should read $-(EIw_{,xx})_{,x} + \overline{P}w_{,x} = R_0$.

167 There should be a plus at the beginning of the second line of Eq. (85).

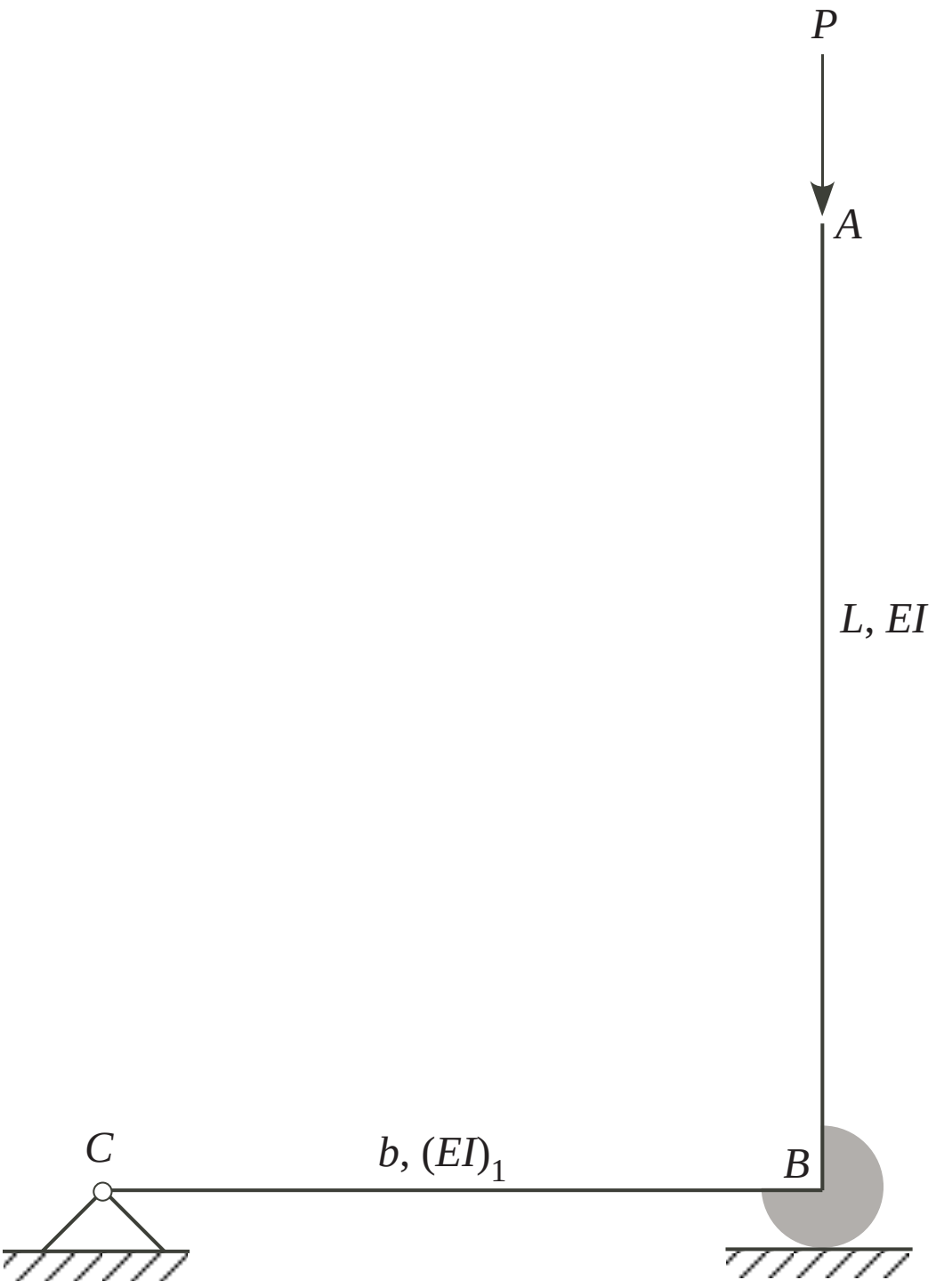

Figure 2: Replacement for Fig. P4.5

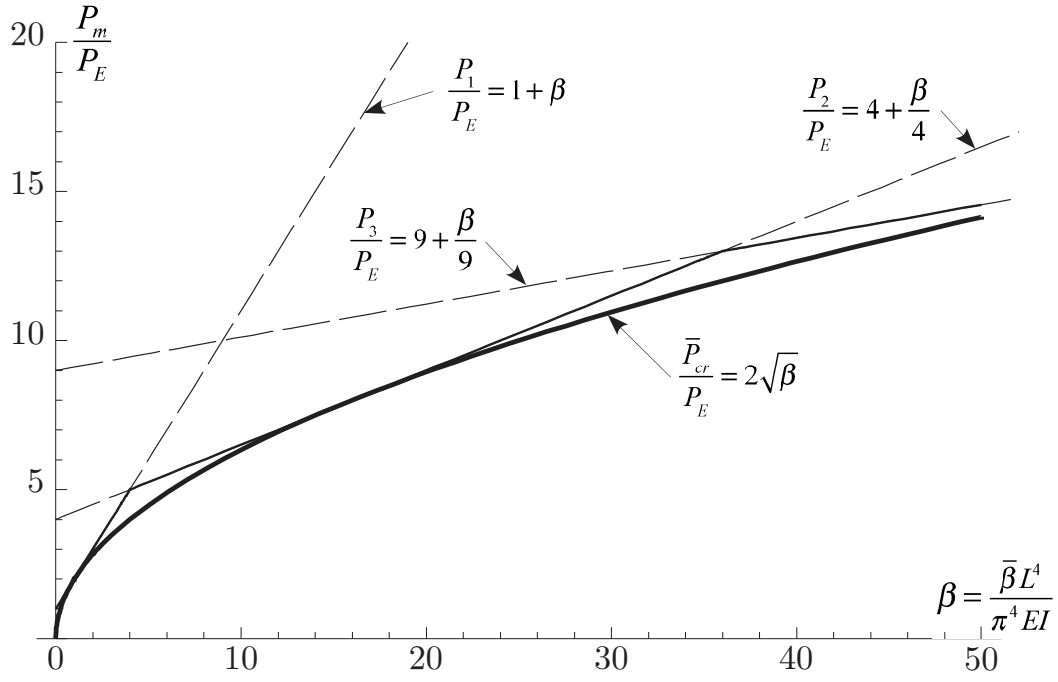

Figure 3: Replacement for Fig. 6.3

170 The stiffness variation should be according to

$$EI(x) = EI_0 \left(1 + \frac{I_1 x}{I_0 L}\right)$$

173 On 10^{th} line from the top, change "pounds per inch (force per length squared)…" to "force per length squared…."

175 Replace Fig. 6.3 with Fig. 3.

176 Append the following to the end of the paragraph after Eq. (10): "It should also be noted from Fig. 6.3 that the piecewise linear curve intersects $2\sqrt{\beta}$ when $\beta = 1, 16, 81, \ldots$, i.e. when $\beta = m^4$ for integer values of m."

183 The large "1" in the numerator of the right-hand side of the unnumbered equation just before Eq. (48) should be sized normally.

194 After Eq. (44), the text should read: "Substitution of Eqs. (44) into Eqs. (34) gives…" (i.e. 34 not 43).

223 In Eq. (180), the last term should be $O\left(\frac{EAh^2\varepsilon^{*2}}{R^2}\right)$

224 In the first of Eqs. (181), the last term should be $O\left(\frac{EAh^2\varepsilon^*}{R^2}\right)$, and the term involving ν should be underlined.

224 In the second of Eqs. (181), the last term should be $O\left(\frac{EAh^3\varepsilon^*}{R^2}\right)$, and the term involving ν should be underlined.

224 The sixth line of text following Eqs. (181) should begin, "$\varepsilon^* << 1$…."

225 The second line of text following Eqs. (190) should read, "… one must restrict $\varepsilon^* = \max(\varepsilon, h\kappa_3)$ to be small…."

229 In Eq. (216), the next to last term on the right-hand side inside the square brackets should read $-\lambda\hat{u}_1''^2$.

231 In Eq. (225), the right-hand side should be multiplied by $(1+\overline{\varepsilon})$.

232 In Eq. (232), M_3 should be $\hat{M}_3$.

232 In Problems 4 and 5, the reference to Section 7.4.2 should be to Section 7.3.2.

235 The line of text just below Eq. (2) should refer to Eqs. (3.118), not Eqs. (3.115).

236 The first line of text should refer to Eqs. (3.118), not Eqs. (3.116).

237 The paragraph just before Eqs. (19) should allude to Eqs. (3.140), not Eqs. (3.137).

237 In the second of Eqs. (19), the extra right parenthesis should be removed.

239 The text just after Eq. (26) should read, "and $R(\theta)$ being the matrix in Eq. (3.124). Thus,…."

243 The first line of Section 8.3.4 should read, "Regarding the direction cosines $C_{12} \approx \hat{u}_2'$ and $C_{13} \approx \hat{u}_3'$ as small perturbation…."

244 Just below Eq. (39), a $\pm$ is missing from 2.86059π.

244 Just before Eqs. (42), the expression for $\mathbf{M} = Q\mathbf{b}_1$ should begin with $Q(\mathbf{B}_1 + \ldots)$ not $Q(\mathbf{B} + \ldots)$.

245 The first *two* sentences immediately following Eq. (49) should be replaced by these *three*: "The critical roots for q are plotted against α in Fig. 8.4 for $-\pi/2 \leq \alpha \leq \pi/2$. Note the special symmetry reflected in the plot: $q(\alpha) = -q(-\alpha)$. At $\alpha = -0.824843$, the nondimensional critical load is 0.915531π whereas…."

246 Fig. 8.4 should be replaced by Fig. 4.

247 The u_2^2 and u_3^2 in Eq. (59) should be $\hat{u}_2^2$ and $\hat{u}_3^2$.

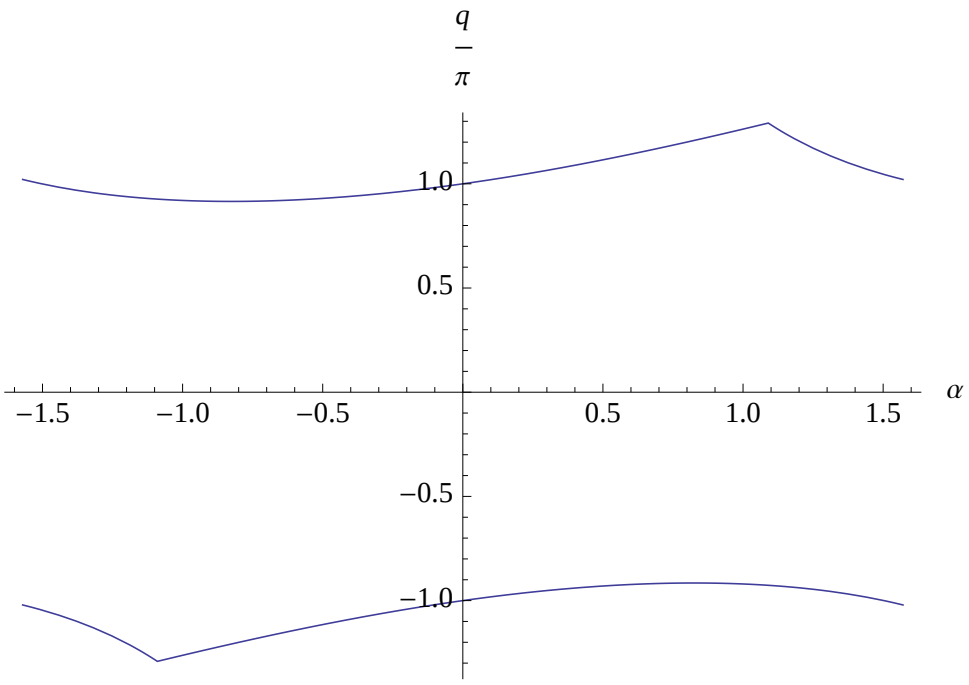

Figure 4: Replacement for Fig. 8.4

247 The first sentence that starts after Eq. (59) should read, "For example, application of Hamilton's principle to a shaft with the same boundary conditions as treated in Section 8.4.3 but with rotation, leads to a weak form. . . ..."

247 Fig. 8.5 caption should read, "Buckling boundary for a clamped shaft loaded by a semi-tangential twisting moment of magnitude qEI/ℓ and an axial compressive force pEI/ℓ^2."

248 Problem 2 should be shortened to read, "Determine and plot the mode shape for problem 1."

248 Problem 5 should be shortened to read, "Show that the last example in Section 8.4.2, when worked using the intrinsic equations instead of by the energy approach, still leads to a characteristic equation of the form of Eq. (49). Find the critical twisting moment for $\alpha = -1$ and $\alpha = \pi/2$, and show that the critical twisting moment versus α behaves as depicted in Fig. 8.4.

249 Problem 7 should read, "... and the $x_1 = \ell$ end free to displace but loaded. . . ."

253 Replace all of Section 9.1.2 with the following:[1] The Vlasov effect stems from the effect of warping rigidity, generally appropriate only for thin-walled beams with open cross sections. In Section 3.8.1 section constants for thin-walled, open-section, isotropic beams are derived;

[1]The second author is grateful to Prof. Noël Challamel for pointing out the error in this section.

see Eqs. (3.172). The consequence of introducing the Vlasov effect in the geometrically exact equations, Eqs. (3.117), is simply a matter of introducing the warping stress resultant

$$Q_1 = E\Gamma\kappa_1' \tag{12}$$

and everywhere in the first two moment equations replacing M_1 with $M_1 - Q_1'$; see Eqs. (50) below. The resulting equations, when perturbed about the prebuckling state for the problem under consideration, reduce to an expanded form of Eqs. (6), viz.,

$$\begin{aligned} \hat{M}_1' + Q\left(\frac{1}{EI_2} - \frac{1}{EI_3}\right)\hat{M}_2 - \frac{E\Gamma}{GJ}\hat{M}_1''' &= 0 \\ \hat{M}_2' - Q\left(\frac{1}{GJ} - \frac{1}{EI_3}\right)\hat{M}_1 - \frac{Q}{EI_3}\frac{E\Gamma}{GJ}\hat{M}_1'' &= 0 \end{aligned} \tag{13}$$

We differentiate the first of Eqs. (13) once with respect to x_1 and use the second to eliminate $\hat{M}_2'$ to obtain a single equation in $\hat{M}_1$ given by

$$-\frac{E\Gamma}{GJ}\hat{M}_1'''' + \left[1 + \frac{Q^2}{EI_3}\frac{E\Gamma}{GJ}\left(\frac{1}{EI_2} - \frac{1}{EI_3}\right)\right]\hat{M}_1'' + \beta^2\hat{M}_1 = 0 \tag{14}$$

where β^2 is given by Eq. (8). Since $\hat{M}_2(0) = \hat{M}_2(\ell) = 0$, it is clear from the first of Eqs. (13) that

$$\hat{M}_1'(0) - \frac{E\Gamma}{GJ}\hat{M}_1'''(0) = \hat{M}_1'(\ell) - \frac{E\Gamma}{GJ}\hat{M}_1'''(\ell) = 0 \tag{15}$$

For the other boundary condition we may either choose to set the warping displacement (proportional to $\hat{M}_1$) or the warping stress resultant (proportional to $\hat{M}_1'$) equal to zero. The simpler solution is the latter, which results in

$$\hat{M}_1'(0) = \hat{M}_1'''(0) = \hat{M}_1'(\ell) = \hat{M}_1'''(\ell) = 0 \tag{16}$$

Thus, a function of the form

$$\hat{M}_1 = a\cos\left(\frac{\pi x_1}{\ell}\right) \tag{17}$$

with a arbitrary satisfies the governing equation and all boundary conditions. Substitution of this expression into Eq. (14) yields a characteristic equation of the form

$$-\frac{E\Gamma}{GJ}\frac{\pi^4}{\ell^4} + \left[1 + \frac{Q^2}{EI_3}\frac{E\Gamma}{GJ}\left(\frac{1}{EI_2} - \frac{1}{EI_3}\right)\right]\frac{\pi^2}{\ell^2} + Q^2\left(\frac{1}{EI_2} - \frac{1}{EI_3}\right)\left(\frac{1}{GJ} - \frac{1}{EI_3}\right) = 0 \tag{18}$$

Values of Q satisfying this equations are the critical loads, for which the solutions are

$$Q_{\text{cr}} = \pm\frac{\pi}{\ell}\sqrt{\frac{GJ\,EI_2\left(1 + \frac{\pi^2 E\Gamma}{\ell^2 GJ}\right)}{\left(1 - \frac{EI_2}{EI_3}\right)\left[1 - \frac{GJ}{EI_3}\left(1 + \frac{\pi^2 E\Gamma}{\ell^2 GJ}\right)\right]}} \tag{19}$$

As expected, this correction raises the critical torque at which buckling occurs. The size of the correction is strongly dependent on the cross-sectional configuration.

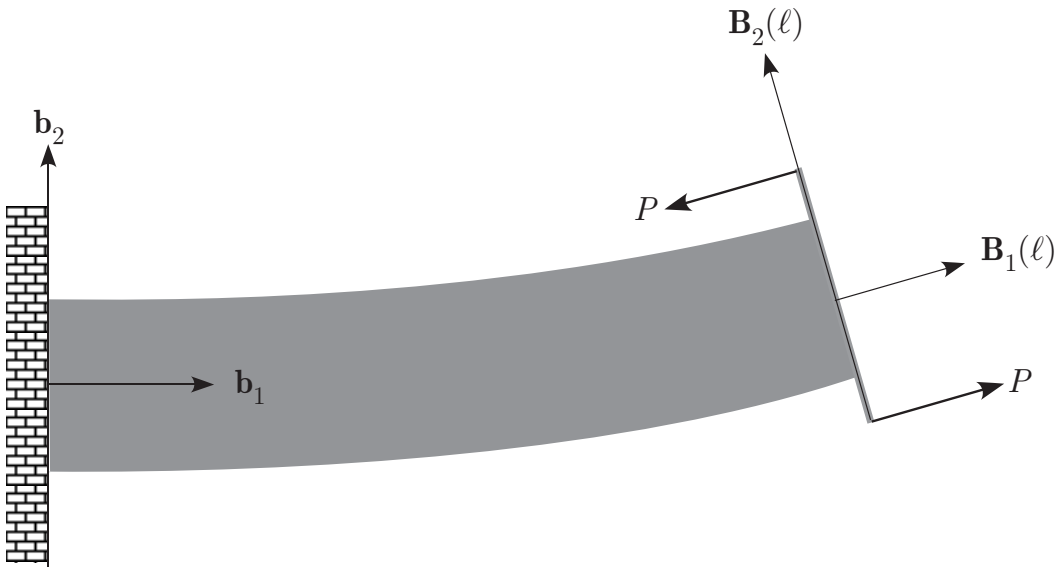

Figure 5: Replacement for Fig. 9.2

255 The line of text immediately following Eqs. (22) should end with $\hat{F}_3 = 0$, not $\hat{F}_1 = 0$.

255 The subscript 3 on the cancelled $\hat{F}_3$ at the end of the second of Eqs. (23) should be moved closer to the F so as to look like other subscripted variables.

255 Eq. (28) should read

$$Q_{\text{cr}} = Q_0 \left[\pm 1 + \frac{(A + B - 2\,A\,B)\,\alpha}{2\pi\sqrt{AB\,(1-A)\,(1-B)}} \right]$$

256 Fig. 9.2 should be replaced by Fig. 5.

257 The first two sentences after Eqs. (35) should be replaced by one: "Because $\hat{M}_1 = \hat{M}_2 = 0$ at the end where the load is applied, one finds that only the trivial solution exists, so that buckling is not possible."

257 The last line on the page should refer to Eqs. (3.140) instead of Eqs. (3.137).

260 Fig. 9.4 should be replaced by Fig. 6.

262 At the very top of the page, the text should start out "where $\overline{\kappa}_3 = \overline{\theta}_3'$."

262 In the second of Eqs. (54), the third term, which presently reads $\overline{M} M_3 \hat{\kappa}_2$ should instead read $\overline{M}_3 \hat{\kappa}_2$.

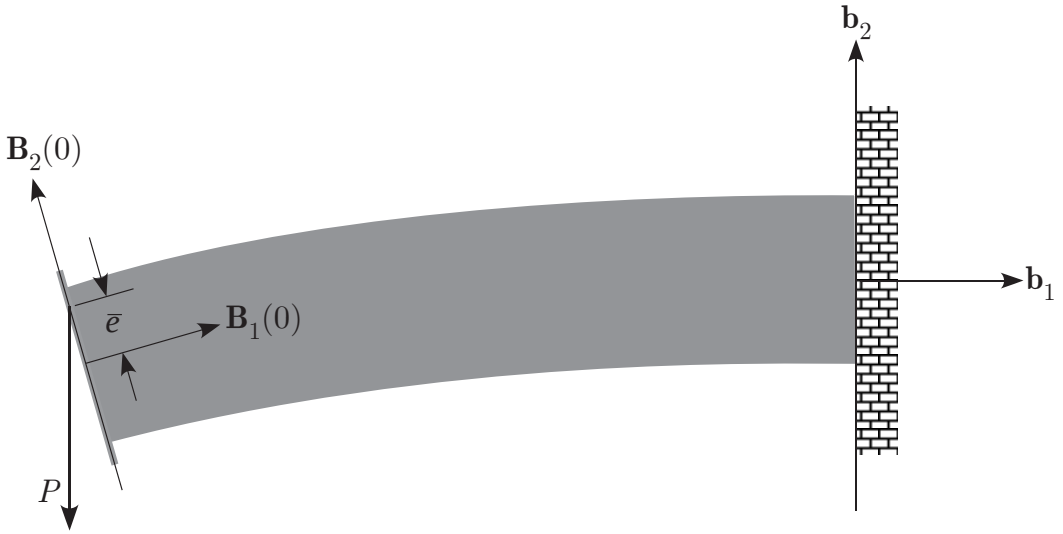

Figure 6: Replacement for Fig. 9.4

262 In the second line of text immediately following Eq. (54), the $e\hat{F}_3(0)$ should be replaced by $\overline{e}\hat{F}_3(0)$.

262 The first line of Eq. (56) should read

$$\max\left(\overline{\theta}_3\right) = \frac{P\ell^2}{2D_{33}}$$

262 Equation (57) should read

$$\overline{\theta}_3 = \frac{P(\ell^2 - x_1^2)}{2D_{33}}$$

263 The first term in the numerator of the second of Eqs. (60), written as $-B\kappa$, should instead be $-Bk$.

276 The problem statement for Problem 2 should start out, "For an initially curved strip-beam as formulated in section 9.1.3. . . ."

276 In Problem 3, omit the parenthetical quantity $\beta/4.01260$.

276 Replace Problem 6 with the following: Consider a deep beam with rectangular cross section, which is attached to spherical bearings at each end that are free to move closer to each other and loaded at the $x_1 = \ell$ end by a semi-tangential bending moment of magnitude Q so as to bend the beam in the plane of its largest flexural rigidity. Let the undeformed beam lie in a plane parallel to the $\mathbf{a}_1$-$\mathbf{a}_2$ plane. Ignoring pre-buckling curvature and rotation (but not bending moment!), determine the buckling load. Discuss the implications of your results.

276 Replace Problem 7 with the following: Consider a deep beam with rectangular cross section, which is attached to a fixed hinge at $x_1 = 0$ and to a spherical bearing at the $x_1 = \ell$ end. The $x_1 = \ell$ end is free to move closer to the $x_1 = 0$ end, and the beam is loaded at the $x_1 = \ell$ end by a quasi-tangential bending moment of magnitude Q so as to bend the beam in the plane of its largest flexural rigidity. Let the undeformed beam lie in a plane parallel to the $\mathbf{a}_1$-$\mathbf{a}_2$ plane. Ignoring pre-buckling curvature and rotation (but not bending moment!), determine how the buckling load depends on the angle α which defines the direction of the forces comprising the quasi-tangential moment. Discuss the implications of your results.

290 The second term in Eq. (45) should be divided by two, so that the equation becomes

$$\delta \int_0^\ell \left[\overline{F}_1 \overline{u}_1' - \frac{\mu \Omega^2}{2} \left(-R + x_1 + \overline{u}_1 \right)^2 \right] dx_1 = 0$$

291 In the beginning of the second line before Eq. (54), the text should say "now become…" instead of "how become…."

291 The first term under the integral in Eq. (54) should be $EI_2 \hat{u}_3''^2$.

295 Problem 3 should end with "…to that of Problem 2."

297 The sentence starting on line 11 and the two following it should read, "Then let q_1 move from zero to $q_1 = \hat{q}_1$. During this motion, the work done by P is $P\ell(1 - \cos\hat{q}_1)\cos\hat{q}_2 - P\ell \sin\hat{q}_1 \sin\hat{q}_2 \approx P\ell(\hat{q}_1^2/2 - \hat{q}_1\hat{q}_2)$. So, the total work done to get in this first way from $q_1 = q_2 = 0$ to $q_1 = \hat{q}_1$ and $q_2 = \hat{q}_2$ is approximately $P\ell(\hat{q}_1^2/2 - \hat{q}_1\hat{q}_2)$.

299 The reference to Section 2.2 just before Eq. (1) should be to Section 2.1.

300 The first of Eqs. (3) should read

$$T = m\ell^2 \dot{q}_1^2 + \frac{m\ell^2}{2} \left(\dot{q}_2^2 + 2\dot{q}_1\dot{q}_2 \right)$$

300 The first matrix in Eq. (5) should read

$$\begin{bmatrix} 2m\ell^2 & m\ell^2 \\ m\ell^2 & m\ell^2 \end{bmatrix}$$

300 The first matrix in Eq. (7) should read

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

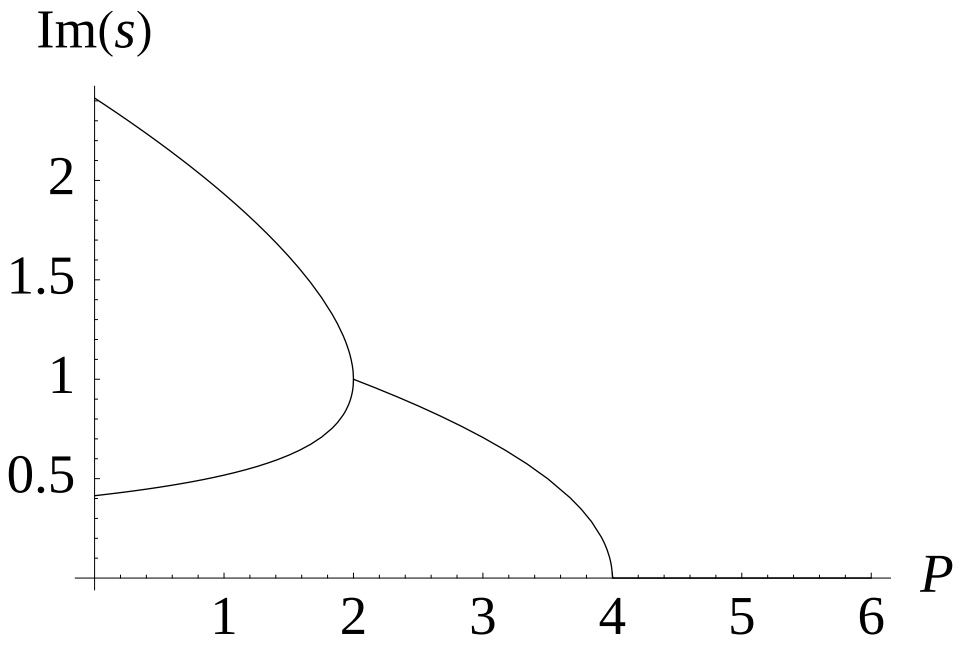

Figure 7: Replacement for Fig. 11.4

300 Eq. (8) should read

$$\begin{vmatrix} 2s^2 + (c_1 + c_2)s + 2 - P & s^2 - c_2 s + P - 1 \\ s^2 - c_2 s - 1 & s^2 + c_2 s + 1 \end{vmatrix} = 0$$

300 Eq. (9) should read

$$s^4 + 2(3 - P)s^2 + 1 = 0$$

301 Eq. (10) should read

$$s^2 = P - 3 \pm \sqrt{(P-2)(P-4)}$$

301 Figs. 11.4 and 11.5 should be replaced as Figs. 7 and 8.

302 Figs. 11.6 and 11.7 should be replaced as Figs. 9 and 10.

305 The ℓ's in the column matrix on the far left-hand side of Eq. (37) should be replaced by 1's.

309 In the second line of Eq. (56), the first of the two occurrences of dx_1 should be changed to $d\xi$.

313 Just after Eq. (74) and after "where" insert: "…the matrix R is defined in Article 3.7.3 and…."

313 The expression just before Eq. (77) showing $s = \mu\overline{s}^2\ell^4/EI$ should read $s^2 = \mu\overline{s}^2\ell^4/EI$.

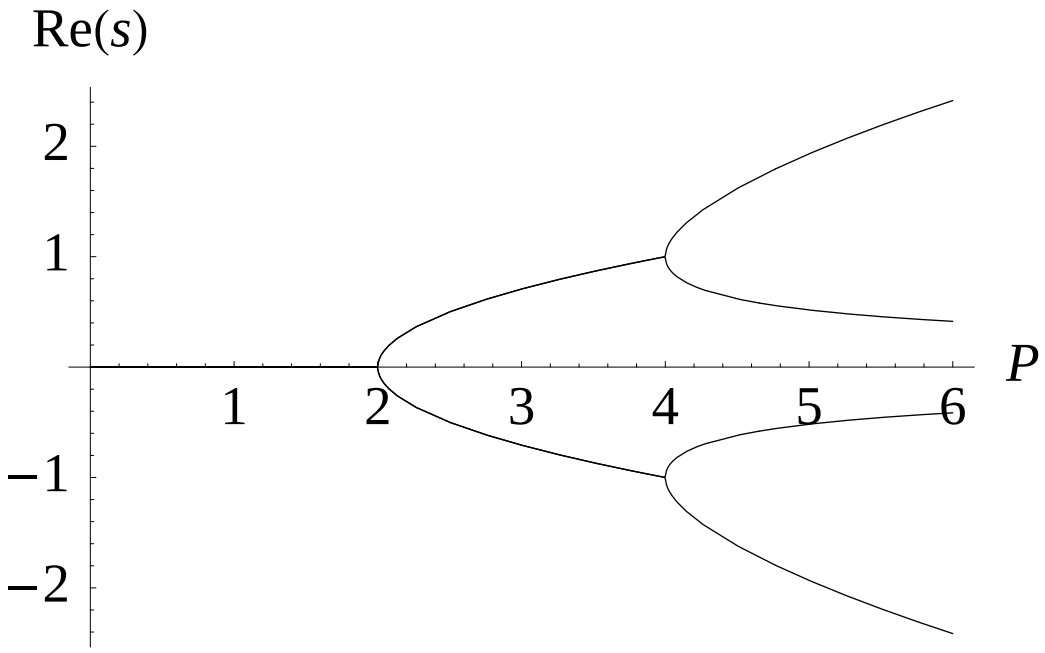

Figure 8: Replacement for Fig. 11.5

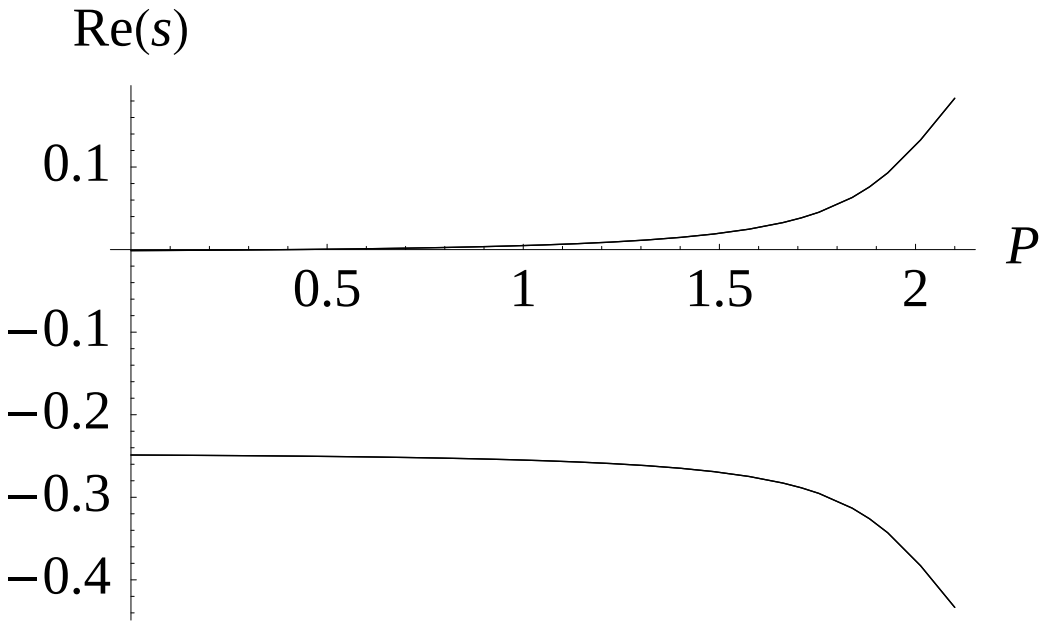

Figure 9: Replacement for Fig. 11.6

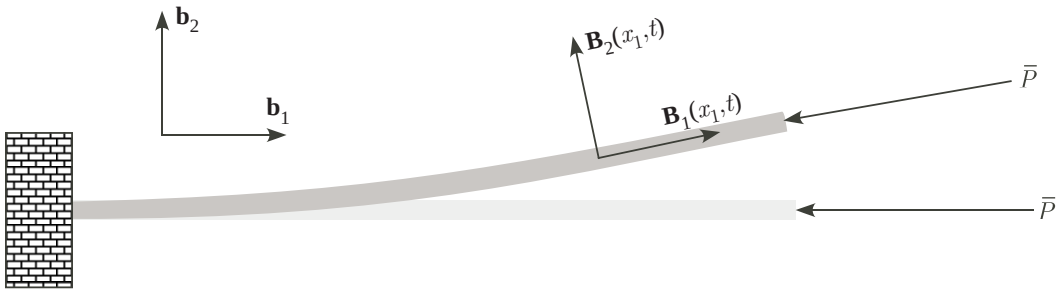

Figure 10: Replacement for Fig. 11.7

315 The equation referred to above Eq. (84) should be Eq. (3.140), not Eq. (3.137).

317 Replace the last sentence just before Article 11.6.2 begins with: "The dimensionless parameter r is the ratio of the fundamental bending and torsion frequencies of the unloaded beam with $e = 0$, and $\beta_1 = 1.87510$ and $\gamma_1 = \pi/2$ are the separation constants for a clamped-free beam undergoing free-vibration in bending and torsion, respectively."

323 The I in the lower-right corner of the square matrix on the right-hand side of Eq. (98) should be i (to distinguish the cross-sectional inertia matrix from the identity matrix). To the text below this equation append "and i is the 3×3 cross-sectional inertia matrix."

326 Problem 2 should read, "Determine the effect of viscous damping on the stability of a clamped-free shaft. . . ."

327 In Problem 4, the quantity $P(\ell - x_1)$ should be $P(1 - x_1/\ell)$.

327 In Problem 5, the word "loaded" is misspelled as "lodaded" and the quantity $\overline{M}_1(\ell)$ should instead be $\overline{M}_3(\ell)$. The next to last sentence should read: "Using Eqs. (115) together with a Galerkin approximation with one term for each of the unknowns, determine the value of the force at which flutter occurs."

327 In Problem 6, the last sentence should read: "Using Eqs. (115) together with a Galerkin approximation with one term for each of the unknowns, determine the value of the moment at which flutter occurs."