Errata for Nonlinear Composite Beam Theory

by Dewey H. Hodges March 1, 2012

Page Description

- 4 The citation Hegemier and Nair (1976) should be Hegemier and Nair (1977).
- 13 The citation Popescu and Hodges (1999b) should be Popescu and Hodges (2000).
- 39 The sentence that starts on line 7 of the last full paragraph should read "Now, γ is the difference between s' times the unit tangent of the deformed beam, 'pulled back' to the frame b orientation by premultiplication with $\underline{C}^{\text{bB}}$, and the unit tangent of the undeformed beam."
- 50 In the last sentence of the second paragraph following Eqs. (3.81), should read, "The former is the local rotation, and the latter is the global rotation."
- 50 The second sentence in the third paragraph following Eqs. (3.81), should read, "Thus, we have $\phi_2 = -x_2\kappa_1$, making it..."
- 60 In the second of Eqs. (4.2), v should be a.
- 66 All \boldsymbol{B}_i on this page should be \boldsymbol{T}_i .
- 67 All \boldsymbol{B}_i on this page should be \boldsymbol{T}_i .
- 67 Eq. (4.26), inside the square brackets of the first line, the term $-w_2/R$ is missing, so it should read

$$\boldsymbol{G}_1 = \left[1 + \overline{\gamma}_{11} - x_2 \left(\frac{1}{R} + \overline{\kappa}_3\right) - \frac{w_2}{R} + w_1'\right] \boldsymbol{B}_1 + \left(w_2' + \frac{w_1}{R}\right) \boldsymbol{B}_2 + w_3' \boldsymbol{B}_3$$

69 In the first line of Eq. (4.34), there should be a 2 in front of the first term inside the angle brackets, so as to read

$$2\mathcal{U}_{1}^{*} = EA\overline{\gamma}_{11}^{2} + EI_{3}\overline{\kappa}_{3}^{2} + 2E\left\langle\frac{x_{2}}{R}\left(\overline{\gamma}_{11} - x_{2}\overline{\kappa}_{3}\right)^{2} - \frac{w_{2}}{R}\left(\overline{\gamma}_{11} - x_{2}\overline{\kappa}_{3}\right)\right\rangle$$
$$+ O\left(\frac{EAa^{2}\hat{\varepsilon}^{2}}{R^{2}}\right)$$

- 70 On second line, "The two underlined terms..." should read "The two underlined terms of Eq. (4.34)"
- 72 Eqs. (4.41) should read

$$E = \left\langle \left\langle \left[\Gamma_{a} S \right]^{T} \mathcal{D} \left[\Gamma_{a} S \right] \right\rangle \right\rangle \qquad D_{a\epsilon} = \left\langle \left\langle \left[\Gamma_{a} S \right]^{T} \mathcal{D} \left[\Gamma_{\epsilon} \right] \right\rangle \right\rangle \\D_{aR} = \left\langle \left\langle \left[\Gamma_{a} S \right]^{T} \mathcal{D} \left[\Gamma_{R} S \right] \right\rangle \right\rangle \qquad D_{al} = \left\langle \left\langle \left[\Gamma_{a} S \right]^{T} \mathcal{D} \left[\Gamma_{\ell} S \right] \right\rangle \right\rangle \\D_{\epsilon\epsilon} = \left\langle \left\langle \left[\Gamma_{\epsilon} \right]^{T} \mathcal{D} \left[\Gamma_{\epsilon} \right] \right\rangle \right\rangle \qquad D_{RR} = \left\langle \left\langle \left[\Gamma_{R} S \right]^{T} \mathcal{D} \left[\Gamma_{R} S \right] \right\rangle \right\rangle \\D_{ll} = \left\langle \left\langle \left[\Gamma_{\ell} S \right]^{T} \mathcal{D} \left[\Gamma_{\ell} S \right] \right\rangle \right\rangle \qquad D_{R\epsilon} = \left\langle \left\langle \left[\Gamma_{R} S \right]^{T} \mathcal{D} \left[\Gamma_{\epsilon} \right] \right\rangle \right\rangle \\D_{l\epsilon} = \left\langle \left\langle \left[\Gamma_{\ell} S \right]^{T} \mathcal{D} \left[\Gamma_{\epsilon} \right] \right\rangle \right\rangle \qquad D_{Rl} = \left\langle \left\langle \left[\Gamma_{R} S \right]^{T} \mathcal{D} \left[\Gamma_{\ell} S \right] \right\rangle \right\rangle$$

- 72 Because $\Psi^T D_{a\epsilon} = 0$, Λ in Eq. (4.46) can be shown to be equal to zero.
- 80 In the first lines of Eqs. (4.91) and (4.92), the quantity YN^{-1} should instead be Y^TN^{-1} . The second line of Eq. (4.92) should read

$$+DN^{-1}\left[(YG^{-1}D_2 - D_3)A_3 - A_3G^{-1}(D_1G + D_2Y)\right]$$

- 81 The QQ^T on the second line of Eq. (4.106) should not be there.
- 81 There should be a minus sign in front of the G_1 in Eq. (4.108).
- 81 Eq. (4.109) should read

$$Y_{1} = AQD_{1}^{T} + BD_{2}^{T} + Y_{0}G_{1}G_{0}^{-1} + CA^{-1}(Y_{0}G_{0}^{-1}D_{2} - D_{3})QG_{0} - AQG_{0}^{-1}P^{T}QG_{0} + \left[A\left(Y_{0}G_{0}^{-1}D_{2} - D_{3}\right)^{T} - \left(BD_{2}^{T} + AQD_{1}^{T}\right)Q^{T}\right]A^{-1}DA^{-1}QG_{0}$$

- 82 The second line should refer to Eq. (4.99), not Eq. (4.94).
- 83 The next to last sentence before Eq. (4.114) should be replaced by: "Similarly, the shear center is the point in the cross-section at which a shear force induces no twist. In an isotropic beam it is the same as the center of twist, the point about which the cross-section rotates under a pure twisting moment."

85 Eq. (4.119) should read

$$w(x_1, x_2, x_3) = S(x_2, x_3) \left[(\hat{V}_0 + V_{1R}) \bar{\epsilon} + V_{1S} \bar{\epsilon}' \right]$$

85 Eq. (4.120) should read

$$U_i(x_1, x_2, x_3) = u_i(x_1) + x_\alpha [C_{\alpha i}(x_1) - \delta_{\alpha i}] + C_{ji}(x_1) w_j(x_1, x_2, x_3)$$

- 85 Insert these two sentences right after the sentence that ends "... frame of the deformed beam" just below Eq. (4.120): "The C in Eq. (4.120) is always C^{Tb} . In the generalized Timoshenko theory one may note that $C = C^{\text{Tb}} = C^{\text{TB}}C^{\text{Bb}}$, where C^{TB} is calculated from (4.67)."
- 86 The next to last sentence before Eq. (4.125) should read: "One may discard κ (effectively replacing K with k) and γ terms in \mathcal{R} by virtue of the small strain approximation."
- 87 The two appearances of x before Eq. (4.127) and the one appearance after Eq. (4.129) should be x_1 .
- 87 The third sentence in the last complete paragraph should read, "In other words the generalized shear center is at $\xi_2 = -\Phi_{34}/\Phi_{44}$ and $\xi_3 = \Phi_{24}/\Phi_{44}$; and when the reference line is placed at that point, then $\Phi_{24} = \Phi_{34} = 0$."
- 90 In Eq. (4.136), κ_2 in the (2,1) element should be κ_3 , and a minus sign should be added in front of x_3 in the (2,2) and (3,2) elements.
- 89 Eqs. (4.132) and (4.134), Γ_w should be Γ_a .
- 93 In Eq. (4.156), a minus sign should be added in front of both occurrences of x_3 .
- 98 Eq. (4.179) should read

$$U_i(x_1, x_2, x_3) = u_i(x_1) + x_\alpha [C_{\alpha i}(x_1) - \delta_{\alpha i}] + C_{ji}(x_1) w_j(x_1, x_2, x_3)$$

106 Eq. (5.7) should read:

$$\widetilde{\overline{\delta\psi}}' = -\delta C' C^T - \delta C C'^T$$

108 The second of Eqs. (5.22) should read:

$$\widetilde{\Omega} = -\dot{C}C^T + C\widetilde{\omega}C^T$$

115 In Eq. (5.50), the second line on page 115, there are two occurrences of m, both of which should be μ , so that the second line reads:

$$-\delta V^{T} \left[\mu \left(V - \widetilde{\overline{\xi}} \Omega \right) - P \right] - \delta \Omega^{T} \left(i\Omega + \mu \widetilde{\overline{\xi}} V - H \right)$$

- 119 The term P/GK_2 in the second part of Eqs. (5.67) should be removed.
- 123 In Eq. (5.92) there is a +m missing from the terms in parenthesis, so that the equation should read:

$$F = F_1 e_1 + \frac{\widetilde{e}_1}{s'} \left(M' + \widetilde{K}M + m - \dot{H} - \widetilde{\Omega}H - \widetilde{V}P \right)$$

130 Equation (5.119) should read

$$\Omega = \Omega_1 e_1 + \frac{\widetilde{e}_1 \left(V' + \widetilde{K} V \right)}{s'}$$

- 178 In the last equation of Eqs. (6.98), each subscripted 2 outside the parentheses should be ,2 instead.
- 218 Two terms are missing from the displacement field given in Eq. (7.25). The equation for u_2 should have the additional term $-x_1x_3\frac{M_1}{GJ}$, and the equation for u_3 should have the additional term $x_1x_2\frac{M_1}{GJ}$.
- 221 In the first term on the right-hand side of Eq. (7.37), the 2 should go inside the anglebracket so as to read

$$2\mathcal{U}_2 = E \left< 2V_1'(\gamma_{11} + x_3\kappa_2 - x_2\kappa_3) + \hat{w}_1^2 \right> + \dots$$

225 In Eqs. (7.57), the expression for q should read

$$q = 4x_2^2 \left[(4+\nu)\rho^2 + 2 - \nu \right] - 12x_3^2 \left[(2-\nu)\rho^2 + \nu \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^4 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^4 + \mu^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^4 + \mu^2 + \nu(13\rho^4 + 2\rho^2 + 1) \right] - 3b^2 \left[16\rho^4 + 8\rho^4 + \mu^2 +$$

- 273 The last paragraph, first line, should read, "... to be addressed...."
- 276 Fig. A.3 should be replaced by Fig. 1 herein.
- 282 The last sentence before the epilogue should read as follows: The values are listed for each Gaussian point identified by its location as: " x_2 , x_3 , σ_{11} , σ_{12} , σ_{13} , σ_{22} , σ_{23} , σ_{33} ," where σ_{ij} are the components of the recovered 3-D stress tensor at this Gaussian point.
- 296 The Hegemier and Nair (1976) citation should be Hegemier and Nair (1977), and the volume and year should be volume 15 and 1977, respectively.
- 297 The Popescu and Hodges (1999b) citation should be changed to Popescu and Hodges (2000), and the year should be changed to 2000.



Figure 1: VABS layup convention