Georgia Institute of Technology
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AE 6220: Rotorcraft Dynamics and Aeroelasticity
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A detailed derivation of the structural terms in the equations of motion presented in Ormiston and Hodges (1972) is not found in the literature. Here we give an outline of how to proceed with such a derivation, based on potential energy.

The potential energy in terms of the two flap angles β_h and β_b and the two lead-lag angles ζ_h and ζ_b can be written as

$$P = \frac{1}{2}\left(K_{\zeta h}\zeta_h^2 + K_{\beta h}\beta_h^2 + K_{\zeta b}\zeta_b^2 + K_{\beta b}\beta_b^2\right) \tag{1}$$

Blade motion, however, is given only in terms of the blade orientation angles ζ and β given by

$$\begin{aligned} \zeta =& \zeta_h + \zeta_b \cos\theta - \beta_b \sin\theta \\ \beta =& \beta_h + \zeta_b \sin\theta + \beta_b \cos\theta \end{aligned} \tag{2}$$

Eq. (2) can be solved for ζ_b and β_b, and the result can be subsituted into the potential energy yielding an expression of the form

$$P = P(\zeta, \beta, \zeta_h, \beta_h) \tag{3}$$

Since the generalized inertia forces for the blade can be written entirely in terms of ζ and β, one can introduce

$$\begin{aligned} K_{\zeta_b} &= \frac{K_\zeta}{R}; \qquad K_{\zeta_h} = \frac{K_\zeta}{1-R} \\ K_{\beta_b} &= \frac{K_\beta}{R}; \qquad K_{\beta_h} = \frac{K_\beta}{1-R} \end{aligned} \tag{4}$$

and set

$$\frac{\partial P}{\partial \zeta_h} = \frac{\partial P}{\partial \beta_h} = 0 \tag{5}$$

yielding a potential energy of the form

$$P = \frac{1}{2}\left\{\begin{matrix}\zeta \\ \beta\end{matrix}\right\}^T \begin{bmatrix} \frac{K_\zeta - R(K_\zeta - K_\beta)\sin^2\theta}{\Delta} & \frac{R(K_\zeta - K_\beta)\sin 2\theta}{2\Delta} \\ \frac{R(K_\zeta - K_\beta)\sin 2\theta}{2\Delta} & \frac{K_\beta + R(K_\zeta - K_\beta)\sin^2\theta}{\Delta} \end{bmatrix} \left\{\begin{matrix}\zeta \\ \beta\end{matrix}\right\} \tag{6}$$

where

$$\Delta = 1 + \frac{R(1-R)(K_\zeta - K_\beta)^2 \sin^2\theta}{K_\zeta K_\beta} \tag{7}$$

These operations can be conveniently carried out with computerized symbolic manipulation software, such as *Mathematica*.

As explained by Ormiston and Hodges (1972), when $R = 0$ there is no elastic coupling, and when $R = 1$ there is “full” elastic coupling.